Superconductivity

①

We know that when temperature of a conductor is decreased, then its resistance decreases linearly according to the relation

$$R = R_0(1+\alpha t) \quad — ①$$

where R_0 = resistance at 0°c
R = resistance at t°c
α = temperature coeff of resistance

This is +ve for metals.
That if temp. is increased then resistance of conductor will also increase and vice versa.

If the temp. is expressed in absolute unit (K) then above equation becomes a straight line passing through origin like $R = R_0 \gamma T$ — ②

Accordingly resistance of a conductor will become zero only at absolute zero (that is 0 Kelvin).

This property was being verified by scientist 'Kamerlingh Onnes' for mercury (Hg).
He was decreasing temp. of Hg and observing its resistance.

②

Then he plotted graph between resistance of Hg and absolute temp.

As expected, the graph was linear and its extrapolation was hinting that graph will pass through origin.

However, surprisingly at temp. 4.18K (≈ 4.2K), the resistance of Hg suddenly became zero as shown in fig.1.

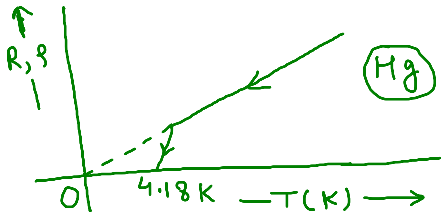


Fig 1: Variation of Resistance (R) or resistivity (ρ) with temp. for Hg

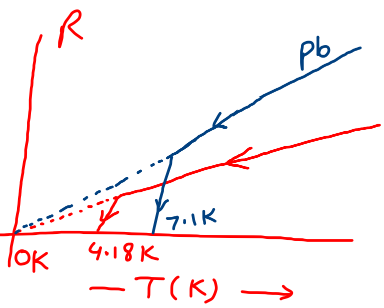
He called this phenomenon as "Superconductivity".

Definition:– The loss of resistance by certain materials when cooled below a specific temp. is called Superconductivity and materials showing this property are called superconductors.

③

Critical temp (T_c)

The minimum temp. to which a material must be cooled so that its resistance becomes zero is called critical temp.



For Hg ; $T_c = 4.18\,K \approx 4.2K$ ✓
For Pb ; $T_c = 7.1K$ ✓

Observation:– Good conductors are not Good superconductors ⇒ Why?

Around 1960-1970

Experimental observation

Model building

Out of Hg & Pb, which is better superconductor

Ans: Pb (lead)

Q? out of Hg & Pb, which is better conductor

Ans: .... ?

Today we have superconductors for which T_c is as high as 135 K. These are called high temp super conductors. 4

These materials are alloys CuO ⇒ Garnets Ln & Ac

Good conductors are not good superconductors.

On what factors T_c depend?

Ans:- It depends on nature of material

Isotopic Effect:- Hg has two isotopes Hg^{198}, Hg^{200}

∴ $T_c = 4.18K$ was for more abudant isotope (Hg^{198})

It was observed that T_c of Hg^{200} was smaller than T_c of Hg^{198}. 5

"The variation of T_c for different isotopes of a single element is called Isotopic effect."

It was experimentally found that if M_1, M_2 are masses of two isotopes of a single element & T_{c_1}, T_{c_2} are their critical temperatures respectively then:

$$T_c \times \sqrt{M_1} = T_{c_2} \times \sqrt{M_2}$$

$P_1V_1 = P_2V_2$
$PV = const$

In general $T_c\sqrt{M} = const$

$$\Rightarrow \boxed{T_c \propto \frac{1}{\sqrt{M}}} \quad \text{—(1)}$$

∴ The critical temp of an element is inversely proportional to square root of its isotopic mass. 6

But isotopic effect has given physicists a hint to understand the reason or cause of superconductivity because isotopic effect is also obeyed by Debye's temp. (T_D) i.e. $T_D \propto \frac{1}{\sqrt{M}}$ is a well known effect before superconductivity

lattice vibration

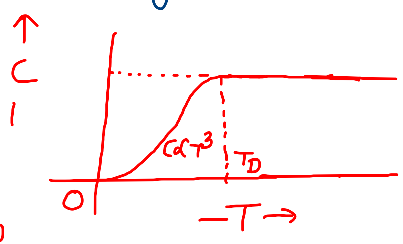
Now we have

$T_c \propto \frac{1}{\sqrt{M}}$

$T_D \propto \frac{1}{\sqrt{M}}$

$\Rightarrow \frac{T_c}{T_D} = const$

$\Rightarrow T_c \propto T_D$



C is independent of temp

$U = N_0 6 \times \frac{1}{2} kT$
$= 3 N_0 k T$
$= 3RT$

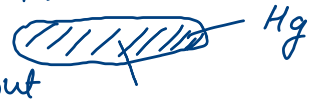
$C = \frac{dU}{ndT} = \frac{dU}{dT} = 3R = const$

(7)

∴ Interrelation of T_c & T_D tells us that one possible reason for superconductivity could be lattice vibration. And it later on turned out to be true. The famous theory of superconductivity was BCS theory. And three scientists Bardeen, Cooper, Sheiffer got noble prize for their work on superconductivity. Till date this theory is most successful in explaining superconductivity.

Effect of magnetic field on super conductivity:- (8)

Under absence of any external mag. field Hg sample will be superconductor if temp is less than T_c ($\approx 4.2K$)



But it is observed that without increasing temp we can destroy superconductivity by applying external magnetic field.

$T = 3\,K$

For Hg $T_c \approx 4.2K$

"Minimum amount of magnetic field that must be applied across a superconducting material to destroy its superconductivity is called critical magnetic field (H_c)"

(9)

It is further observed that if we have two samples of Hg one at temp 3K & other at 2K (Note that both temp. are smaller than T_c (4.2K)), stronger magnetic field was required to destroy superconductivity of sample at 2K than sample at 3K. Thus max. external field is required at absolute zero for a given sample.

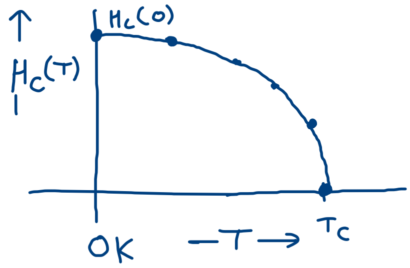
Conclusion:- Critical mag field H_c for a given material is a function of temp. and it will decrease as T will increase.

$$\therefore \; H_c = f(T)$$

$$H_c = \max \text{ when } T = 0\,K$$

$$H_c = 0 \text{ (min) when } T \geq T_c$$

Experimentally H_c was plotted as function of T 10



and variation of H_c VS T for Hg & most of other materials was a parabolic curve as shown in the figure.

Empirically relation between H_c & T was found to be as follows:

$$\boxed{H_c(T) = H_c(0)\left[1 - \frac{T^2}{T_c^2}\right]} \quad \text{— (1)}$$

where

Critical current (I_c) :— we know that superconductivity 11 can be destroyed by magnetic field.

We also know that when current is passed through a wire/conductor then it produces its own magnetic field (Oersted's experiment)

∴ Even if no external field is applied, then current passing through conductor will produce mag. field. As current is increased, field also increases. A stage will reach when field produced due to current will become equal to critical field H_c and superconductivity gets destroyed.

The current passing through superconductor at 12 this stage is called critical current (I_c).

" The maximum current that can be passed though a superconductor without destroying its superconductivity is called critical current".

or

" The minimum current that must be passed through a superconductor to just destroy its superconductivity is called critical current".

Chapter Videos

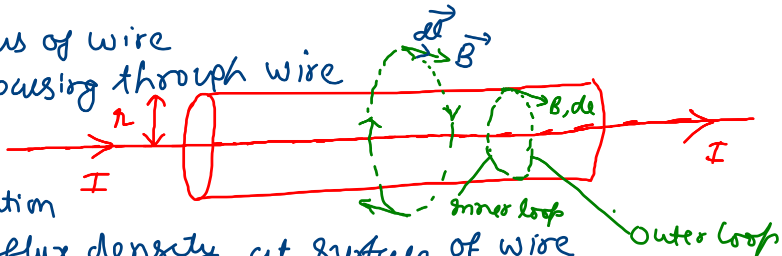
t.ly/sdalf

t.ly/nxhp

Expression for I_c ; Silsbee's Rule :- (13)

Let r = radius of wire

I = current passing through wire



B = mag. induction or mag. flux density at surface of wire

The field B will make circular loops according to RHTR. We move on this loop in the same sense as magnetic field so that angle b/w $\vec{B}$ & $\vec{dl}$ is zero everywhere along the loop. (Since B is max at surface ∴ we take radius of loop = r)

---

(14)

By Ampere's circuital law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\oint B\,dl\cos 0^\circ = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

$$\Rightarrow \frac{B}{\mu_0}(2\pi r) = I \quad \text{—(1)}$$

But $\frac{B}{\mu_0} = H$

∴ (1) becomes

$$H(2\pi r) = I \quad \text{—(2)}$$

Let H_c = critical field

∴ By definition

$H \leq H_c$ for material to be superconductor

$$\Rightarrow 2\pi r H \leq 2\pi r H_c$$

$$\Rightarrow I \leq 2\pi r H_c \quad \text{(using)}$$

∴ I_c = max allowed current

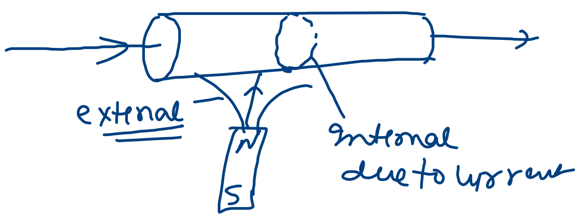
$$\boxed{I_c = 2\pi r H_c} \quad \text{—(3)}$$

---

(15)

Eq. (3) gives critical current in the absence of external magnetic field.

In the presence of external mag. field (H) I_c decreases and new formula becomes

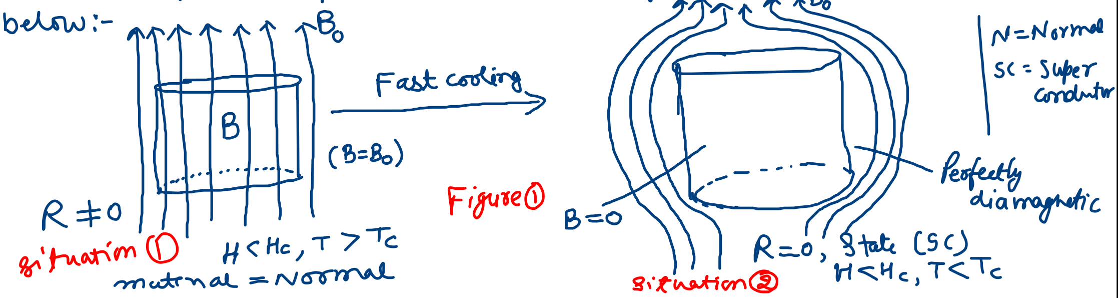


$$\boxed{I_c = 2\pi r (H_c - 2H)} \quad \text{—(4)}$$ where H = ext. mag. field

Equation (4) is called Silsbee's rule.

Chapter Videos
t.ly/sdalf
t.ly/nxhp

Flux expulsion or Meissner effect :- When a superconducting material is placed in longitudinal magnetic field ($< H_c$) and temp of material is above T_c. Now the material is cooled at a fast rate so that temp. drops below T_c, then it is found that material becomes superconductor and magnetic field lines are pushed out of the material. This process is called flux expulsion or Meissner effect. It is shown diagrammatically below:- (16)



∴ We can say from this observation that whenever a material becomes superconductor, it simultaneously becomes perfectly diamagnetic. (17)
∴ Meissner effect is also called Perfect diamagnetism of superconductors.

It is clear from above experimental observation that when material becomes superconductor, magnetic induction takes place inside it. It is obvious that induced magnetic induction is equal and opposite to applied magnetic induction.

Let $\vec{B_0}$ = applied magnetic induction
$\vec{B_M}$ = Induced magnetic induction (magnetisation)
$\vec{B}$ = Net magnetic induction

By law vectors $\vec{B} = \vec{B_0} + \vec{B_M}$ —①

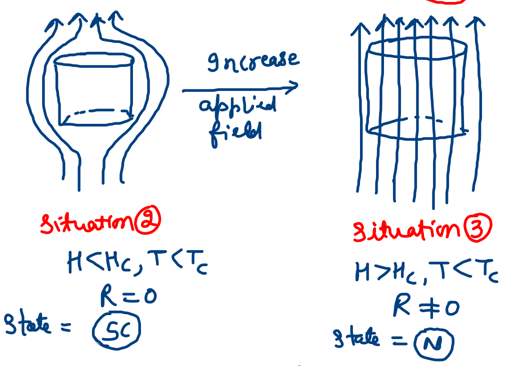
Also from experimental observation $\vec{B} = 0$

∴ ① becomes $0 = \vec{B_0} + \vec{B_M}$

$\Rightarrow \quad \vec{B_M} = -\vec{B_0}$ —②

∴ If we increase B_0, then B_M will also increase in opposite direction so that B remains zero.

But we can do this only upto critical field. Beyond that material will become normal conductor & field lines will again start passing through material & material will loose its perfect diamagnetism. This is shown diagrammatically in figure ②. (18)



So we can say that we can expect B_M to increase only upto the level of critical field. When applied field becomes more than critical then B_M vanishes.

We know that if

$\vec{M}$ = magnetic dipole moment per unit volume (magnetisation vector)

χ_m = magnetic susceptibility

μ_0 = permeability of free space

H = magnetic field (magnetising force)

∴ We know that:

$$\vec{B}_m = \mu_0 \vec{M}$$

$$\vec{B}_0 = \mu_0 \vec{H}$$

Put these values in (2)

19

$$\therefore \quad \mu_0 \vec{M} = -\mu_0 \vec{H}$$

$$\Rightarrow \quad \vec{M} = -\vec{H} \quad \text{——} (3)$$

We also know that

$$\chi_m = \frac{M}{H} \qquad \left(\chi_m = \frac{I}{H}\right)$$

$$\Rightarrow \quad M = \chi_m H$$

In Vector form

$$\vec{M} = \chi_m \vec{H} \quad \text{Put in (3)}$$

$$\therefore \quad \chi_m \vec{H} = -\vec{H}$$

$$\Rightarrow \boxed{\chi_m = -1} \quad \text{——} (4)$$

∴ Mag. susceptibility of SC is always −1.

Equation (3) in magnitude can be written as follows:

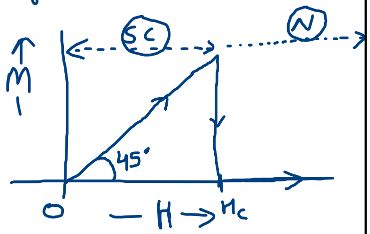
$$M = H \quad \text{——} (5)$$

($y = mx$ is a st. line passing through origin)

∴ If graph is plotted b/w M & H then it should be a straight line passing through origin and its slope $m = \tan\theta = 1$ and the line will make angle 45° with both axes. Moreover H can be increased only upto Hc.

20

∴ This whole observation can be represented in the form of figure as follows:-



Superconductors are non ohmic conductors:- (By using Meissner effect)

21

We know from Maxwell's equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{——} (1)$$

From ohm's law in vector form

$$\vec{J} = \sigma \vec{E} \quad \text{——} (2)$$

Suppose superconductors obey ohm's law (2). But for superconductors Resistivity $= \rho = 0$

Now in eq. (2)

$$\sigma = \text{Conductivity} = \frac{1}{\rho}$$

Put in (2)

$\Rightarrow$ we get

$$\vec{J} = \frac{1}{\rho} \vec{E}$$

$$\Rightarrow \vec{E} = \rho \vec{J}$$

But $\rho = 0$

This gives $\vec{E} = 0$

Put in (1)

$$\vec{\nabla} \times 0 = -\frac{\partial \vec{B}}{\partial t}$$

$$0 = -\frac{\partial \vec{B}}{\partial t}$$

If we integrate wrt time

then $\vec{B}$ = constt.

∴ when material becomes superconductor then magnetic flux density within the superconductor should remain constant.

However observation made in Meissner effect is exactly opposite.

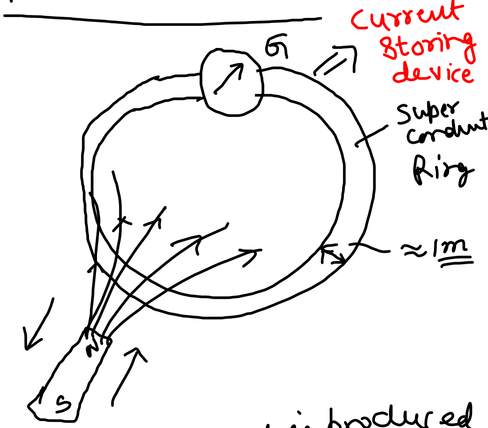
(22)

That is when ρ becomes zero inside material/superconductor then B also becomes zero.

Hence assumption made by us is wrong.

Hence superconductors donot obey ohm's law and these are non-ohmic superconductors.

Persistent Currents:-



When current is produced in a superconducting ring using Faraday's laws of em induction.

(23)

The current produced in the superconducting ring did not die out even for a year.

It was observed that current can continue to pass through superconducting ring for thousands of years before dying out.

"This phenomenon of no loss of current in a superconducting ring is called Persistent current"

Even this phenomenon indicates that superconductors donot obey ohm's law.

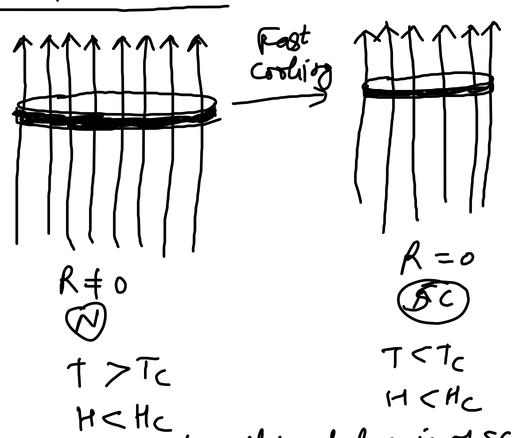
Hg — conductor $T > 4.2K$; Superconductor $T < 4.2K$

Whenever a substance is termed as superconductor it always means that we have kept

$T < T_c$
$H < H_c$
$I < I_c$

Flux penetration:-

(24)



When Temp of very thin film is of SC decreased below Tc then field lines are not expelled out of material.

That is these lines continue to pass through the material. This phenomenon is called "Flux penetration".

Conclusion:– Meissner effect is only shown by thick superconductors. Thin films of superconductors don't show Meissner effect.

25

Type-I and Type-II Superconductors:–

It was predicted from observation of Meisser effect that graph between M & H will be a straight line passing through origin and this graph will extend upto H_c.

However when this fact was being verified for different materials, then two kinds of variation graphs b/w M & H

26

were observed as shown in figures below:-

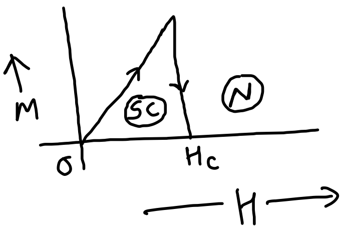
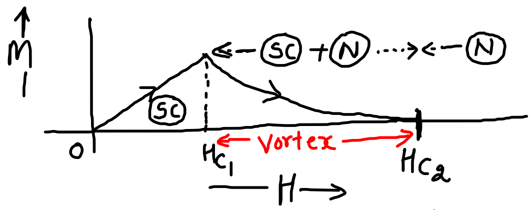


Figure – 1

Type-I superconductor
or
Soft superconductor
or
Ideal superconductor

Pure Elements — More sensitive to field



Type-II Superconductor
or
Hard superconductors
$(H_{c_2} \approx 100 H_c)$
or
Real superconductor

Alloys — less sensitive to field

27

Many materials (mostly elements) obey Meissner effect very strictly and have only one critical field. When applied is just increased beyond H_c, then whole of material looses superconductivity all of a sudden. These materials are not commercially important although these are ideal superconductors and obey Meissner effect perfectly.

However many other materials (alloys of metals & nonmetals) obey Meissner effect only upto certain field H_{c_1} (called first critical field). Beyond H_{c_1} superconductivity starts decreasing slowly and it is completely destroyed at very higher value H_{c_2} called second critical field. From 0 to H_{c_1} material is perfectly superconductor from H_{c_1} to H_{c_2} (called vortex state) material is partially SC & partially N. Beyond H_{c_2} superconductivity is completely destroyed.

London Equations:–

These are equations developed by two brothers F. London & H. London, which helped to develop mathematical model for explaining certain properties of superconductors.

Let m = mass of an electron
$-e$ = charge on electron
$\vec{v}$ = drift velocity of electrons
$\vec{J}$ = current density
A = area of cross section of superconducting rod.
n = no. of superconducting electrons (i.e. electrons which travel through lattice without suffering any collision) per unit volume

28

In a normal conductor, electrons move under the effect of applied voltage (or electric field) and these electrons suffer resistive force because of collision

$\therefore$ mass × acc. = Electric force + Resistive force

(Newton's 2nd law)

$$\Rightarrow m\vec{a} = -e\vec{E} + \vec{F_R} \quad \text{—①}$$

For a superconductor $\vec{F_R} = 0$

$\therefore$ equation ① becomes

$$m\vec{a} = -e\vec{E}$$

$$\Rightarrow \vec{a} = \frac{-e\vec{E}}{m} \quad \text{—②}$$

Also expression for drift velocity is given by

$$v = \frac{I}{nAe}$$

$$= \frac{J}{ne} \qquad \left(\because \frac{I}{A} = J\right)$$

In vector form

$$\vec{v} = -\frac{\vec{J}}{ne} \quad \text{—③}$$

29

Negative sign indicates that current direction & $\vec{v}$ direction are opposite.

Differentiate equation ③ w.r.t. 't'

$$\Rightarrow \frac{d\vec{v}}{dt} = -\frac{1}{ne}\frac{d\vec{J}}{dt}$$

$$\Rightarrow \vec{a} = -\frac{1}{ne}\frac{d\vec{J}}{dt}$$

Put in ②

$$\Rightarrow -\frac{1}{ne}\frac{d\vec{J}}{dt} = -\frac{e\vec{E}}{m}$$

$$\Rightarrow \boxed{\frac{d\vec{J}}{dt} = \frac{ne^2}{m}\vec{E}} \quad \text{—④}$$

London's 1st eqn.

30

Take curl on both sides of eqn. no ④

$$\Rightarrow \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{J}) = \frac{ne^2}{m}(\vec{\nabla} \times \vec{E})$$

$$= \frac{ne^2}{m}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\left(\because \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow \frac{\partial}{\partial t}\left\{\vec{\nabla} \times \vec{J} + \frac{ne^2\vec{B}}{m}\right\} = 0$$

Integrate both sides w.r.t. 't' assuming that when $t = 0$, then $\vec{J}, \vec{E}, \vec{B}$ were zero.

$$\Rightarrow \vec{\nabla} \times \vec{J} + \frac{ne^2}{m}\vec{B} = 0 \quad \text{—⑤}$$

We also know that

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

Where $\vec{A}$ is called Vector potential

Put in ⑤

$$\Rightarrow \vec{\nabla} \times \vec{J} + \frac{ne^2}{m}\vec{\nabla} \times \vec{A} = 0$$

$$\text{or} \quad \vec{\nabla} \times \left\{\vec{J} + \frac{ne^2}{m}\vec{A}\right\} = 0$$

$$\therefore \quad \vec{J} + \frac{ne^2}{m}\vec{A} = 0$$

$$\text{or} \quad \boxed{\vec{J} = -\frac{ne^2}{m}\vec{A}} \quad \text{—⑥}$$

London's second equation

There is another approach to simplify equation (5).

We know that

$$\vec{\nabla} \times \vec{B} = \mu_0 (\vec{J})$$

where $\vec{J}$ is total current density

$$\Rightarrow \quad \vec{J} = \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B})$$

Put in (5)

$$\frac{1}{\mu_0} \{ \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) \} + \frac{ne^2}{m} \vec{B} = 0$$

$$\Rightarrow \quad \underbrace{\vec{\nabla}(\vec{\nabla} \cdot \vec{B})}_{0} - \nabla^2 \vec{B} + \frac{ne^2 \mu_0}{m} \vec{B} = 0$$

or

$$-\nabla^2 \vec{B} + \frac{ne^2 \mu_0}{m} \vec{B} = 0$$

(31)

or

$$\boxed{\nabla^2 \vec{B} = \frac{ne^2 \mu_0}{m} \vec{B}} \quad — (7)$$

Equation (7) is also called London's Second equation.

Significance or Importance of London's 1st equation :–

London's 1st equation is $\frac{d\vec{J}}{dt} = \frac{ne^2}{m} \vec{E}$ —(8)

Suppose $\vec{E} = 0$ (when there is no voltage source like cell/battery)

∴ Eq. (8) gives

$$\frac{d\vec{J}}{dt} = 0$$

$$\Rightarrow \quad \vec{J} = \text{constant} \quad \left(\text{although } E = 0\right)$$

∴ If current was zero at $t = 0$ It will remain 0 forever

But if current was having some value at $t = 0$, then this current will continue to pass through circuit forever.

∴ London's 1st equation is successful in explain the phenomenon of persistent currents in superconductors & it indicates that superconductors donot obey ohm's law. ($V = IR$ or $J = \sigma E$)

(32)

Significance of London's 2nd equation

Consider eq. (6)

$$\vec{J} = \underbrace{\frac{-ne^2}{m}}_{\text{Constant}} \vec{A}$$

∴ $\vec{J} \propto \vec{A}$

ohm's law says that $J \propto E$

∴ London's 2nd eqn. gives replacement of ohm's law for superconductors. It tells that current density in a superconductor is directly proportional to vector potential ($\vec{A}$) instead of electric field.

Let us now consider equation (7)

$$\nabla^2 \vec{B} = \frac{ne^2 \mu_0}{m} \vec{B} \quad — (7)$$

We know unit of $\vec{\nabla}$ is m^{-1}

(33)

∴ unit of LHS of (7) is $T\,m^{-2}$

∴ unit of RHS should also be $T\,m^{-2}$

∴ quantity $\frac{ne^2 \mu_0}{m}$ must have unit of m^{-2}

∴ we call $\sqrt{\frac{m}{ne^2 \mu_0}} = \lambda$ —(9)

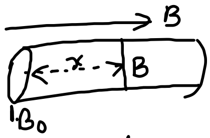
where λ must have unit of length & this quantity is called "Penetration depth".

Chapter Videos
t.ly/sdalf
t.ly/nxhp

We know $\nabla^2 \vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2}$

To have better understanding of eq. (7) we consider only 1-D case

$\therefore \nabla^2 \vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} \equiv \frac{d^2 \vec{B}}{dx^2}$



$\therefore$ (7) simplifies to

$$\frac{d^2 \vec{B}}{dx^2} = \frac{1}{\lambda^2} \vec{B}$$

In scalar form

34

$$\frac{d^2 B}{dx^2} = \frac{1}{\lambda^2} B \quad \text{—— (9)}$$

The solution of eq. (9) is

$$B = B_0 e^{-\frac{x}{\lambda}} \quad \text{—— (10)}$$

where B_0 = magnetic induction at $x = 0$

B = mag. induction at distance x within super conductor

Case I If $x >> \lambda$ (thick SC)

then $\frac{x}{\lambda} \approx \infty$

$\therefore$ From eq. (10)

$$B \approx B_0 e^{-\infty} = \frac{B_0}{e^{\infty}} = \frac{B_0}{\infty}$$

$$B = 0$$

$\therefore$ Magnetic induction will be zero inside superconductors whose thickness/length is very large compared to λ.

35

This tells that thick superconductors will show Meissner effect.

Hence London's 2nd equation successfully explains Meissner effect.

Case II If $x << \lambda$ (thin SC)

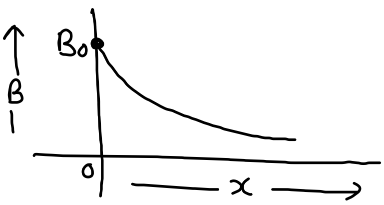
$\therefore \frac{x}{\lambda} \approx 0$

$\therefore$ (10) gives $B \approx B_0 e^{0}$

$\Rightarrow B = B_0$

$\therefore$ Thin superconductors will show flux penetration

Penetration depth:- Variation of B wrt x is as shown as follows



If we put $x = \lambda$

36

then from eq. (10)

$$B = B_0 e^{-1} = \frac{B_0}{e}$$

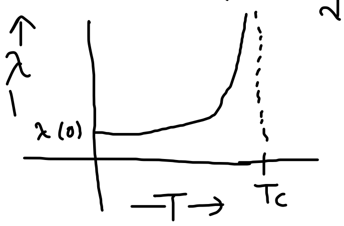
$\therefore$ Penetration depth λ is that thickness of material (SC) upto which magnetic induction decreases to $\frac{1}{e}$ of initial value

(Note $e = 2.7183 \Rightarrow \frac{1}{e} = 0.318$)

$\frac{1}{e} = 31.8\%$

37

Note: Penetration depth of a superconductor depends upon nature of material and on temp of material because of factor n $\left(\lambda = \sqrt{\frac{m}{ne^2\mu_0}}\right)$



The empirical relation for variation of λ wrt temp in Kelvin

$$\boxed{\lambda(T) = \frac{\lambda(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}}$$

Where $\lambda(0)$ = penetration depth at absolute 0 temp.
$\lambda(T)$ = penetration depth at absolute temp. T

At $T=0$ all $\bar{e}$s are SC
$\therefore n = max$
$\therefore \lambda = minimum$

as T is increased no. of SC $\bar{e}$s start decreasing $\therefore \lambda$ starts increasing

At $T = T_c$, $n = 0$
& λ becomes ∞ as shown in figure

38

BCS Theory :-

A solid is composed of +ve ions and valence electrons move through lattice in radom manner (i.e. there is no corelation between motion of different electrons).

Due to this reason, electrons collide with lattice ions and loose their kinetic energy in the form of heat.

Thus, these collisions between electrons and lattice ions, which result in loss of kinetic energy in the form of heat is the cause of resistance.

An example of electron travelling through lattice randomly and loosing kinetic energy at each collision with +ve ions and wasting energy as heat is shown in fig. 1.

39

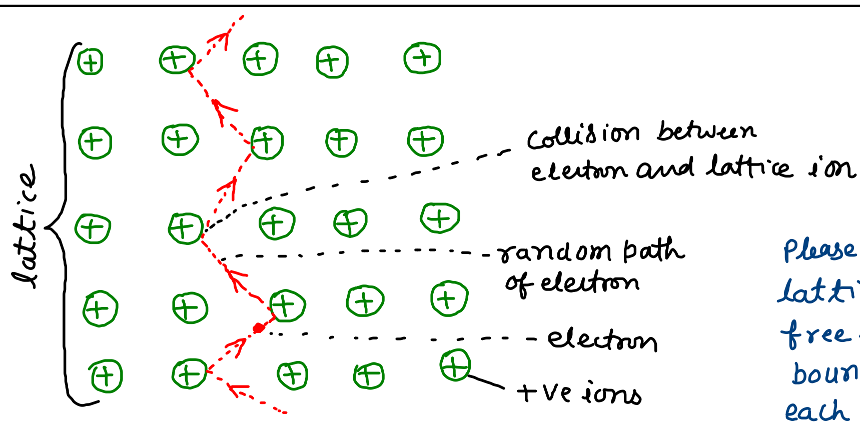


Fig. 1: Random motion of an electron through a lattice

Please remember that lattice ions are not free. These are bound/connected to each other through bonds and energy, which binds them together is called "cohesive energy".

Chapter Videos
t.ly/sdaIf
t.ly/nxhp

40

The quantum theory of superconductivity was given combinedly by three scientists Bardeen, Cooper and Schrieffer in 1957.

This theory is called BCS theory.

According to this theory, "superconductivity is due to attractive interaction between electrons at very low temperature. Due to this interaction, electrons form pairs called Cooper pairs. The two electrons in each cooper pair move coherently through lattice in such a way that they donot suffer any collision with lattice ions. Due to this, resistance becomes zero and material becomes superconductor."

Let us now try to understand BCS theory conceptually (in qualitative manner).

41

Consider an electron e_1 moving with momentum $\vec{p_1}$ through lattice as shown in fig. 2.

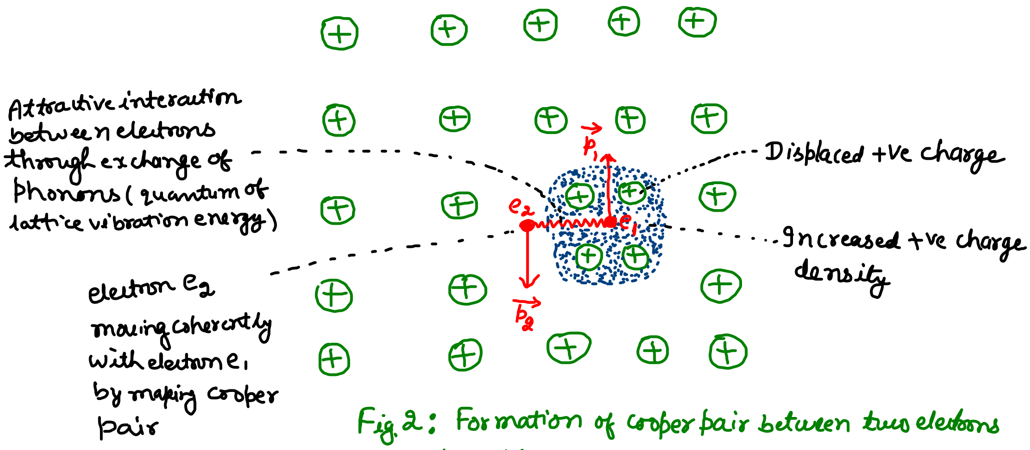


Fig. 2: Formation of cooper pair between two electrons in lattice and moving coherently without suffering any collision.

42

Because of Coulomb interaction, the lattice ions (+vely charged) surronding e_1 will be attracted toward it. The extent, by which +ve ions will be attracted toward electron e_1 depends on cohesive energy of lattice (smaller cohesive energy will result in more displacement of lattice ions and vice versa).

Due to displacement of ions, +ve charge density will increase around electron e_1 (This increased charged density is shown by blue dots in fig. 2).

If by chance, another electron e_2 passes closely to the electron e_1 then:

(i) Due to condensation of ions around e_1, screening effect will increase between e_1 and e_2. That is e_1 will be shielded from repulsive coulomb interaction from e_2. Or we can say that repulsive interaction between e_1 & e_2 in the presence of condensation of +ve ions (increased +ve charge density) around e_1 will be lesser as compared to repulsive interaction when electrons move freely.

(ii) The electron e_2 will be attracted toward electron e_1 due to increased +ve charge density around it. The second electron e_2 prefers to remain in region of increased +ve charge density (which is created by electron e_1 and hence e_2 will follow e_1 or we can say that two electrons will start moving coherently through lattice. It appears as if two electrons are bonded with each other and this pair of electrons, which moves coherently through lattice is called "Cooper Pair". 43

Since both electrons in a cooper pair are moving coherently through the lattice, therefore these electrons donot collide with lattice ions and as a result resistance of material becomes zero and it becomes superconductor. This explains superconductivity.

Hence cause of superconductivity according to BCS theory is as follows: "As the temp. is decreased, +ve charge density starts increasing around every first moving electron, due to which other electron coming from opposite direction is attracted toward first electron. At critical (or lower temp.) attractive interaction is sufficient to form virtual bond between two electrons and cooper pairs are formed. The two electrons in each cooper pair move coherently through the lattice and no collision takes place between electrons and lattice ions. Due to this, resistance of material becomes zero and it becomes superconductor." 44

More discussion on superconductivity

1. We have discussed earlier that motion of electron in lattice results in displacement of lattice ions toward it, which causes increase in +ve charge density. However, electron being very light particle moves very fast as compared to ions. So before the instant that displaced ions trap electron, the electron comes out of this region (shown shaded with blue dots in the figure drawn on page 41). Because of this ions start moving toward their original position. But these lattice ions are already bonded to each other (ionic or covalent bonds) and these bonds behave like spring. This results in setting up of lattice vibrations by the electron. In other words, electron has transferred some energy to lattice, which caused lattice vibrations. However, just like light energy, the lattice vibration energy is also quantized and smallest packet of lattice vibration energy is called "Phonon." 45
Since lattice vibrations are capable of exerting mechanical pressure, so exchange of phonon or lattice vibration energy will always result in exchange of momentum.

Now during formation of cooper pair, one electron caused increase in +ve charge density and other electron will be attracted toward increased +ve charge density. Therefore,

Chapter Videos

t.ly/sdalf

t.ly/nxhp

it is equivalent to say that first electron has transffered phonon or momentum to lattice and second electron has gained this phonon or momentum from lattice. The phonon exchange between two electrons in cooper pair is shown diagramatically as below:- 46

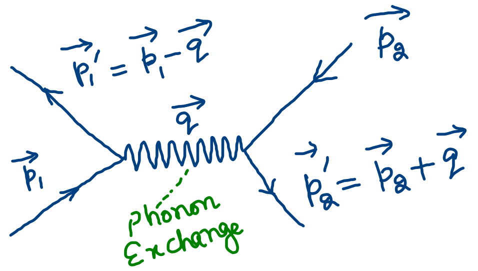


Fig 3: Exchange of phonon between two electrons during the formation of cooper pair

During formation of cooper pair, the montemum of first electron decreases from $\vec{p}_1$ to $\vec{p}_1' = \vec{p}_1 - q$. The decreased momentum $\vec{q}$ is transferred to lattice through phonon. On the other hand, momentum of second electron increases from $\vec{p}_2$ to $\vec{p}_2' = \vec{p}_2 + q$. The additional momentum $\vec{q}$ is given by the lattice to second electron in the form of phonon. Thus phonon exchange between two electrons takes place indirectly through the lattice. It should be noted that in the absence of lattice, the two electrons cannot attract each other two form cooper pair. 47

2. The possibility of attraction and hence formation of cooper pair between two electrons is more if these electrons are moving with high speeds in opposite directions. Because if electron e_1 is fast then it will cause lattice distortion and produce condensed charge condition and quickly comes out of consdensed charge region. On the other hand, due to their large mass and slow speed, the displaced ions in the condensed charge density

will take more time to restore their original positions. 48
Hence region of increased +ve charge density will remain there for sometime. Due to this second electron coming from opposite direction will experience more attractive interaction from first electron through the lattice as compared to direct repulsion between two electrons (coulomb repulsion) and overall interaction between two electrons will be attractive. This condition is feasible for the formation of cooper pair (as shown in Fig 4(a)).

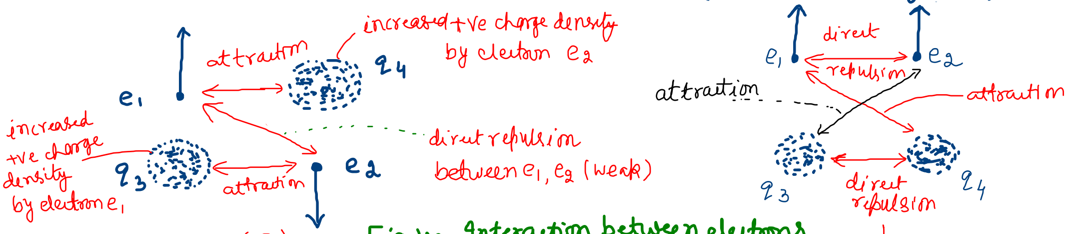


Fig 4: Interaction between electrons when these are moving in (a) opposite direction (b) same direction

However, if electrons are moving in same direction, then direct repulsion between electron-electron and between condensed charge densities will be very large as compared to indirect attraction between electrons through the lattice. Hence formation of cooper pair between electrons moving in same direction is not feasible. (49)

4. Concept of quasi particle. A quasi particle describes a process, which treats elementary excitations in solids (like spin waves) as particles. In other words, a disturbance in a medium, which affects motion of other particles is called quasi particle. The concept of quasi particle is purely a quantum mechanical concept. Phonon, which is quantum mechanical analog of sound/lattice vibrations is an example of quasi particle. A cooper pair in superconductors is also a quasi particle.

4. Let us now write some important characteristics of cooper pairs :- (50)

(i) Two electrons in cooper pair have opposite momenta and opposite spin. Their momenta are represented as $\vec{p}_1 \uparrow$ and $\vec{p}_2 \downarrow$. If magnitude of their momenta are equal (i.e. $p_1 = p_2$), then current density due to such cooper pair will be zero, otherwise (for $p_1 \neq p_2$), it will not be zero.

(ii) The mass of cooper pair is $2m_e^*$, where m_e^* is effective mass of electron in the lattice.

(iii) Net charge on cooper pair is $-2e$; where $e = 1.6 \times 10^{-19} C$

(iv) Net spin of cooper pair is $+\frac{1}{2}\uparrow + (-\frac{1}{2}\downarrow) = 0$, which is an integer. Therefore, a cooper pair is a Boson (i.e. it does not obey Pauli's Exclusion principle).

(v) When cooper pair is formed, then energy of the order of $10^{-4} eV - 10^{-3} eV$ is released. This energy, which is released at the time of formation of cooper pair is called its binding energy ΔE. This binding energy corresponds to temperature range $1K - 10K$ (This can be verified by putting thermal energy kT equal to $10^{-4} eV$ and $10^{-3} eV$). Hence cooper pairs are formed (in most of elemental superconductors) at very low temperature or we can say that superconductivity is a low temp. phenomenon. The empirical formula for binding energy of a cooper pair is given by:- (51)

$$\boxed{\Delta E \approx 3.53\, k T_c \sqrt{1 - \frac{T}{T_c}}} \quad \text{--- (1)}$$

At absolute zero ($T = 0K$) binding energy of cooper pair becomes

$$\boxed{\Delta E \approx 3.53\,kT_c} \quad \text{(at absolute zero)} \text{ —— (2)}$$

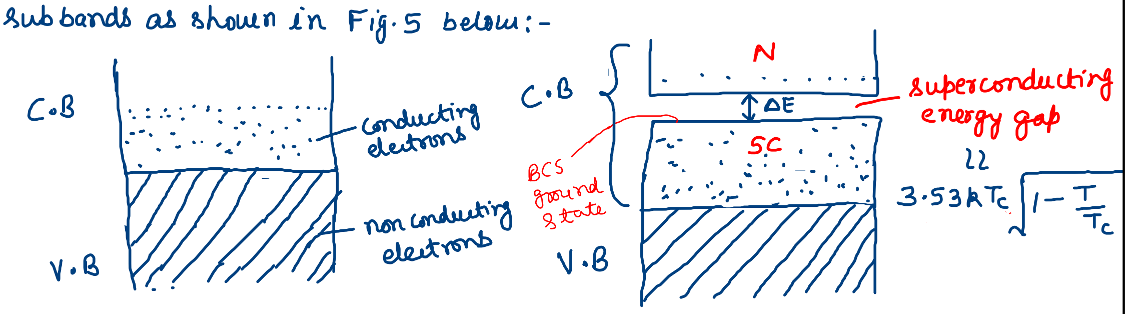
52

In the above expressions k = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K and T_c = critical temperature.

(vi) We know that in any solid substance, only those electrons take part in electrical conduction, which lie in the conduction band. Therefore conducting electrons in a conductor (called normal electrons) or in a superconductor (called superconducting electrons) also belong to conduction band of the material, with only difference that normal electrons in a conductor suffer collisions with lattice ions and lose energy as heat (because these electrons are moving independently in the lattice), while electrons in superconductor donot suffer any collision with lattice ions (because these electrons move coherently in the lattice in the form of Cooper pairs. Hence, when a material

53

becomes superconductor, then its conduction band splits into two sub bands as shown in Fig. 5 below:-



$T > T_c$

(a)

(b)

Fig 5: (a) Energy band diagram of a solid at $T > T_c$ (Normal conductor)
(b) Energy band diagram of a solid at $T < T_c$ (Superconductor)

54

We know (according to BCS theory) that when material becomes superconductor, then Cooper pairs are formed and energy is released. This released energy is actually the binding energy of Cooper pairs and it is also the superconducting band gap energy ΔE between superconducting and normal bands. The expression for ΔE is given by eq. (1) on page 51. It should be noted that when conduction band splits into Normal and Superconducting band, then lower band is superconducting band ($\because$ at $T < T_c$ superconducting states/electrons are more stable than normal states/electrons) and upper part of conduction band will be normal band. Moreover, at absolute zero of temp., all electrons will be superconducting.

Chapter Videos
t.ly/sdalf

t.ly/nxhp

55

As the temperature of material is increased, cooper pairs start breaking (ΔE starts decreasing) and some of the super-conducting electrons become normal conducting electrons (i.e. these again start colliding with lattice ions). Finally at temp. $T \geq T_c$ all cooper pairs are broken (ΔE becomes zero) and hence all electrons become normal conducting electrons.

(vii) Fig. 4(a) describing formation of cooper pairs through contribution of condensed charge densities has been reproduced here. The charges on electrons, condensed charges and the distances between these electrons and condensed charges are shown in the figure.

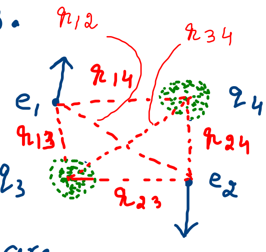


Fig 4(a)

56

The potential energy of this system due to electric forces is given as follows:

$$U = \frac{1}{4\pi\epsilon_0}\left[\underbrace{\left(\frac{e_1 e_2}{r_{12}} + \frac{q_3 q_4}{r_{34}}\right)}_{\text{repulsive term } (U_+)} + \underbrace{\left(\frac{e_1 q_3}{r_{13}} + \frac{e_1 q_4}{r_{14}} + \frac{e_2 q_3}{r_{23}} + \frac{e_2 q_4}{r_{24}}\right)}_{\text{attractive term } (U_-)}\right] \quad (3)$$

It should be noted that e_1 and e_2 are negative ($e_1 = e_2 = -e$; where $e = 1.6 \times 10^{-19}$C), while q_3, q_4 are positive charges. Hence first two terms in equation (3) will contribute +ve potential energy (repulsive interaction), while next four

57

terms in this equation will contribute negative potential energy (indirect attractive interactions). Next important point to note down is the fact that charges e_1, e_2 are fixed charges of electrons, while charges q_3, q_4 increase as the temperature of material is decreased (because these are condensed charges). When $T > T_c$, positive potential energy term (U_+) is much greater than negative potential energy term (U_-) and hence total potential energy will also be +ve due to which cooper pair formation cannot take place. But if temp. of material is decreased, then value of condensed charges (q_3, q_4) start increasing, while e_1, e_2 being absolute charges on electrons, remain same. Hence the value of negative potential energy U_- starts increasing and U_+ remains almost constant on decreasing temperature. If we continue to decrease temperature, then at some specific temp. (T_c), the values of U_+ and U_- become equal and net value of U becomes zero. On decreasing temperature further below T_c, U_- becomes greater than U_+ and overall potential energy U becomes negative, which means that overall attractive interaction between two electrons (through lattice or condensed charges) will be established, leading to formation of Cooper pairs and 58

hence, material becomes a superconductor. 59

(viii) The electrons in a normal conductor move almost indepently of each other. Hence their total energy is almost equal to the sum of kinetic energies of both electrons. If kinetic energy of one electron is $\frac{1}{2}mu^2$, then total energy of a pair of two independently moving electrons will be

$$E_e = \frac{1}{2}mu^2 + \frac{1}{2}mu^2 = mu^2 \quad \text{---} (4)$$

But when cooper pair is formed between two electrons (on decreasing temperature below T_c), then potential energy U (which is negative) gets added into their kinetic energies and total energy of cooper pair will be $E_c = E_e + U = mu^2 + U$ —(5)

Since pot. energy U is negative, so we can put $U = -|U|$, where $|U|$ is +ve and it represents magnitude of attractive potential energy between electrons because of interaction through lattice vibrations. Hence equation (5) can be rewritten as follows: 60

$$E_c = mu^2 - |U| \quad \text{---} (6)$$

The binding energy of cooper pair can also be written as difference between sum of energies of two electrons before and after the formation of Cooper pair. That is

$$\Delta E = E_e - E_c = mu^2 - (mu^2 - |U|)$$

$$\Rightarrow \boxed{\Delta E = |U|} \quad \text{---} (7)$$

Thus binding energy of cooper pair is also equal to magnitude of potential energy of interaction between electrons and condensed charges. 61

(ix) In order to break a Cooper pair energy equal to or more than binding energy of cooper pair (That is $\Delta E = |U| = 3.53\, kT_c \sqrt{1 - \frac{T}{T_c}}$), which is also equal to superconducting energy gap, has to be supplied to the material.

We know the Cooper pair is easily formed between electrons moving with high speeds in opposite directions (see point no. 2 on page 47). But fastest moving electrons in a conductor are those, which lie near the fermi level. Thus most stable cooper pairs are formed from electrons lying very close to fermi level.

The fermi energy of electrons moving in 3-D space with momentum p is given by

$$E_F = \frac{p^2}{2m^*} \quad \text{—— (8)}$$

62

Here m^* is effective mass of electron in the lattice.

But $$p^2 = \left(\frac{h}{\lambda}\right)^2 = \left(\frac{h}{2\pi} \times \frac{2\pi}{\lambda}\right)^2 = (\hbar k)^2 \quad \text{—— (9)}$$

Here $\hbar = \frac{h}{2\pi} = 1.054 \times 10^{-34}\, Js$ is Planck's reduced constant and

$k = \frac{2\pi}{\lambda}$ is propagation constant

However k is a vector quantity. $\therefore \vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$

Hence $k^2 = k_x^2 + k_y^2 + k_z^2$

Put in (9), we get $$p^2 = \hbar^2 (k_x^2 + k_y^2 + k_z^2) \quad \text{—— (10)}$$

Thus equation (8) can be rewritten as

$$E_F = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2 + k_z^2)$$

63

or $$k_x^2 + k_y^2 + k_z^2 = \left(\frac{\sqrt{2m^* E_F}}{\hbar}\right)^2 \quad \text{—— (11)}$$

Equation (11) is the equation of a sphere in the momentum space defined by momentum components k_x, k_y, k_z and radius of this sphere is $\frac{\sqrt{2m^* E_F}}{\hbar}$. Thus Fermi energy level in momentum space is represented by the surface of a sphere having center at origin and radius $\sqrt{2m^* E_F}/\hbar$. This surface is called Fermi surface as shown in Fig. 6 below:-

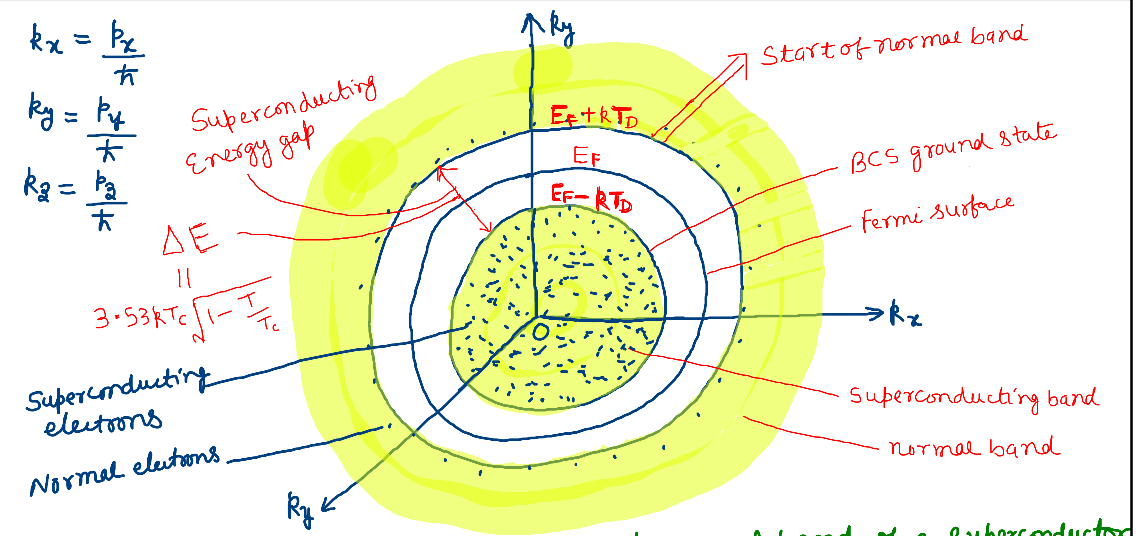


Fig 6:- Fermi surface, superconducting band and normal band of a superconductor in momentum space

Since the origin has coordinates $k_x = k_y = k_z = 0$, which means that $p = \hbar k = 0$ at origin and p increases away from origin in any direction. Thus maximum momentum will be at the fermi surface. The low energy or low momentum region (shown shaded) below E_F is collection of filled energy states with Cooper pairs. The surface of filled energy states below E_F is called BCS ground state. It has been observed that BCS ground state lies below E_F by an energy difference of approximately kT_D, where T_D is called Debye temperature, which is an important parameter for lattice vibrations. 65

Thus radius of BCS ground state is $\approx \frac{\sqrt{2m^* E_F}}{\hbar} - kT_D$

The BCS ground surface represents end of superconducting band in the band diagram of superconductor (see fig. 5(b)). 66

Similarly the normal band in momentum space is also spherical in shape. This band starts above E_F at energy difference approximately equal to kT_D and extends upto ∞ thereafter (theoretically). This means, minimum radius of normal band in momentum space is

$\approx \frac{\sqrt{2m^* E_F}}{\hbar} + kT_c$ and maxium radius is ∞.

Thus in terms of T_D, the minimum gap between normal band and superconding band (which is also the binding energy of cooper pair) is given by $\Delta E \approx (E_F + kT_D) - (E_F - kT_D)$

67

or $\Delta E \approx 2kT_D$ — (12)

But ΔE is also given by $\Delta E = 3.53kT_c\sqrt{1-\frac{T}{T_c}}$

$$\therefore \boxed{2kT_D \approx 3.53kT_c\sqrt{1-\frac{T}{T_c}}} \quad \text{— (13)}$$

Hence Debye temp. and critical temp. are closely related for a superconducting material. Therefore, lattice vibrations play important role in establishing superconductivity in a material at low temp. $T<T_c$ as predicted from property of isotopic effect (see page 6).

There are two important features of BCS ground state:

(a) The energy of BCS ground state is lower than energy of Fermi surface. Thus BCS ground state is more stable than Fermi state.

68

(b) The one particle states are occupied in pairs in momentum space i.e. if a state with wave vector (or propagation vector) with momentum $\vec{k}$ and spin up(↑) is occupied, then state with momentum $-\vec{k}$ and spin down(↓) is also occupied. Conversely, if a state with momentum $\vec{k}$ and spin ↑ is empty, then state with momentum $-\vec{k}$ and spin ↓ will also be empty.

Note:- All spheres in Fig 6 are part of conduction band only, because $k=0$ at origin and $k>0$ for all other points in momentum space, which represents freely moving electrons only and such electrons exist in conduction band in the energy band diagram.

(X) Coherence Length :— After the formation of Cooper pair the two electrons move coherently in the lattice upto certain distance and

69

then Cooper pair breaks. The electrons produced in this manner make new Cooper pairs with other electrons and again move coherently upto certain distance and then again break up and the process continues.

"The average or mean distance traveled by a cooper pair electrons coherently in the lattice from formation to destruction of Cooper pair is called Coherence length." The expression for coherence length (ξ_0) at absolute zero of temp. is given as follows:

$$\boxed{\xi_0 = \frac{\hbar v_F}{2\Delta E}} \quad \text{— (14)}$$

Where v_F = velocity of electrons on Fermi Surface.

i.e. $E_F = \frac{1}{2}m_e^* v_F^2$ or $\boxed{v_F = \sqrt{\frac{2E_F}{m_e^*}}}$

Thus BCS theory has helped to us to understand many properties of superconductivity in conceptual manner.

—x—x—x—

Chapter Videos

t.ly/sdalf

t.ly/nxhp