CURVE TRACING

1) Find the points of inflexion of the curve $y = 3x^4 - 4x^3 + 5$.

2) Trace the curve $r = a(1 + \cos\theta)$.

3) Trace the curve $x = a\cos^3\theta$, $y = b\sin^3\theta$.

4) Trace the curve $x^3 + y^3 = 3axy, a \geq 0$.

PARTIAL DIFFERENTIATION AND ITS APPLICATIONS

1) If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, then use Euler's theorem to prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u$.

2) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that
$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$.

3) Use Lagrange's method to find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$.

4) Use Lagrange's method to find minimum value of $x^2 + y^2 + z^2$,given that
$xyz = a^3$.

5) Expand $x^2y + 3y - 2$ in powers of (x- 1) and (y + 2) using Taylor's theorem.

6) If $Z = f(x+ay) + \emptyset(x - ay)$, Prove that $\frac{\partial^2 z}{\partial y^2} = a^2\frac{\partial^2 z}{\partial x^2}$.

7) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

8) if $u = f(2x-3y, 3y-4z, 4z-2x)$, prove that :
$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$$.

9) If u = f(y-z, z-x, x-y),

Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

10) Expand e^{xy} *up to second degree term at* (1,1).

MULTIPLE INTEGRAL

1) Evaluate the integral $\int_0^1 \int_{x^2}^{2-x} dydx$ by Changing the order of integration .

2) Find , by double integration , the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

3) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

4) Evaluate the integral $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}} dydx$ by changing the order of integration .

VECTOR CALCULUS

1) Use Divergence theorem to evaluate $\iint_S \vec{F}.\vec{dS}$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$; $z = 0$ *and* $z = 3$.

2) Show that the angular velocity at any point is equal to half the curl of linear velocity at that point of the body .

3) Find the direction in which the directional derivative of

$$f(x,y) = (x^2 - y^2)xy \quad \text{at } (1,1) \text{ is zero .}$$

4) Prove that gradient field describes an irrotational motion.

5) Find the directional derivative of $\varphi = e^{2x}cosyz$ at the origion in the direction of the tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$; at $t = \frac{\pi}{4}$

FOURIER SERIES

1) State Dirichlet's conditions for expansion of F(x) in Fourier series.

2) Obtain the Fourier series for the function $f(x) = x^2$, $-\pi \le x \le \pi$.
Hence deduce that: $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} - - - - = \frac{\pi^2}{6}$.

3) Write the function for the Triangular wave form . Also sketch its graph.

4) What is meant by Half- Range series ?

5) Is a function of periodic function periodic , justify ?.
Find the Fourier series to represent $f(x) = x$ when $0 \le x \le 2\pi$.

6) Define a periodic function . What is the period of a constant function ?

7) Find the Fourier series to represent $f(x) = x^2 - 2$ when $-2 \le x \le 2$.

8) Define Saw tooth wave form and find its Fourier series .