

[Total No. of Questions: 9]

[Total No. of Pages: 2]

Uni. Roll No.

Program/ Course: B.Tech.(Sem-1/2)

Name of Subject: Mathematics-II

Subject Code: BSC-104

Paper ID: 15940

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

- 1) Part – A & B are compulsory
- 2) Part- C has two Questions Q8 & Q9 and both are compulsory, but with internal choice.
- 3) Any missing data may be assumed appropriately.

Part – A

[Marks: 02 each]

Q1.

- a) State Dirichlet's conditions for a function $f(x)$ to be expressed as Fourier series.
- b) Find the points of inflexion of the curve $Y = x^3 + 8x^2 - 270x$.
- c) Find the equation of the tangent plane and normal line to the surface $\frac{x^2}{2} - \frac{y^2}{3} = z$, at the point $(2, 3, -1)$.
- d) Evaluate $\int_0^2 \int_1^2 \int_0^{yz} xyz \, dx \, dy \, dz$.
- e) If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, Prove that $\Delta(\log r) = \frac{\vec{r}}{r^2}$.
- f) Show that gradient field describing a motion is irrotational.

Part – B

[Marks: 04 each]

Q2. Obtain the half range sine series for the function $f(x) = x^2$ in the interval $0 \leq x \leq 3$.Q3. Trace the curve $r = a(1 + \sin\theta)$ by discussing its features.Q4. Evaluate the integral $\int_0^1 \int_{4y}^4 e^{x^2} \, dx \, dy$ by Changing the order of integration.

- Q5. If z is a function of x and y and u, v be two other variables such that $u = lx + my, v = ly - mx$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$.
- Q6. Find the smaller of the areas bounded by the ellipse $4x^2 + 9y^2 = 36$ and the straight line $2x + 3y = 6$.
- Q7. If $\vec{F} = \nabla v$, where u, v are scalar fields and \vec{F} is a vector field, show that $\vec{F} \cdot \text{curl } \vec{F} = 0$.

Part - C

[Marks: 12 each]

- Q8 (a) Verify Green's theorem for $\oint_c (x^2 - \cosh y)dx + (y + \sin x)dy$, where c is the boundary of a rectangle whose vertices are $O(0,0), A(\pi,0), B(\pi,1), C(0,1)$.

OR

- Q8 (b) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c$.

- Q9. (a) Expand $f(x) = x \sin x; 0 < x < 2\pi$ as a Fourier Series.

OR

- Q9. (b) Use Lagrange's method to find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$
