Semiconductors (SC)

Fermi Dirac Distribution Function:- The particles which obey Pauli's exclusion principle and are indistinguishable from each other are called Fermions. It has been a well known fact that all half integral spin particles are fermions. Since electrons and holes half integral spins, therefore these particles are Fermions. ~~In order to~~ The distribution of fermions among various energy levels ~~at~~ a given temperature is governed by a probability distribution function called Fermi-Dirac distribution function, which is given by the following expression:-

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}} \qquad\qquad —①$$

This function gives probability that an electron can occupy energy level E at thermal equilibrium. Here E_F is ~~a~~ a reference energy called Fermi Energy and Energy level corresponding to E_F is called Fermi level. The function $f(E)$ is also known as Fermi Factor. Equation ① is also known as Fermi-Dirac equation.

Variation of Fermi Factor:— The fermi-Dirac distribution function given in equation ① above can take values only between 0 and 1. Thus it represents probability or occupancy of energy levels by electrons.

case I   At absolute zero temperature (0K)
consider any energy level corresponding to energy E less than

reference energy E_F (called Fermi-Energy). Therefore $E-E_F$ is negative. Since $T=0K$ ∴ from ① $f(E) = \dfrac{1}{1 + e^{-\infty}} = \dfrac{1}{1+0} = 1$

This means that any level below E_F is completely occupied with electrons at $0K$.

Now consider an energy level having energy E greater than E_F. Now $E-E_F$ is positive. Since $T=0K$ Thus from ① we can write

$$f(E) = \frac{1}{1 + e^{\infty}} = \frac{1}{1+\infty} = 0$$

Thus all energy levels above E_F are completely empty at $0K$.

① when $E = E_F$ we take $\lim\limits_{T \to 0}$ (Rather than $T=0$) Thus we get

$$f(E_F) = \lim_{T \to 0} \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$= \lim_{T \to 0} \frac{1}{1 + e^{0}}$$

$$= \lim_{T \to 0} \frac{1}{1+1}$$

$$= \frac{1}{2}$$

It means that probability of occupancy of Fermi level is $\frac{1}{2}$. That is, it may be a filled level or an empty level. In fact when E_F is a virtual level (Just like pure/intrinsic semi-conductor), then it is completely empty at $0K$ and when E_F is a real level (Just like conductors), then it is completely filled at $0K$. The variation of $f(E)$ at $0K$ for a conductor is shown in Fig(1a) and Fig 1b.
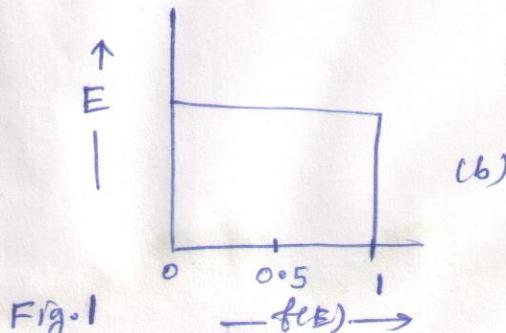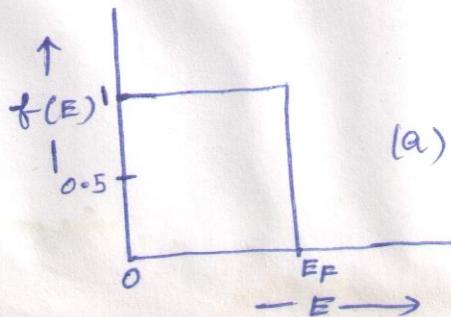

Fig.1

From this discussion, we can define Fermi level as follows:-

"It is a level at OK, below which all existing levels are completely filled with electrons and above which all existing levels are completely empty".

Case II   If temperature is more than absolute zero $(T > 0K)$

When a solid is heated, then electrons start absorbing this heat and transfer to excited states. However the energy given in this manner can be absorbed by electrons which are near the Fermi level E_F only. The electrons, which are in levels well below E_F are not able to excite to higher states.

Consider a state $E > E_F$, so that $E - E_F$ is positive. Since kT is also positive, therefore if $E \to \infty$ then $(E - E_F)/kT \to \infty$

So that $f(E) \to 0$

On the other hand when $E < E_F$ then $E - E_F$ is negative. But $T > 0$ ∴ $(E - E_F)/kT$ is also negative. We assume that temp. T is such that quantity kT is much greater than value of E_F. Under this assumption as $E \to 0$ $(E - E_F)/kT \to -\infty$

So that $f(E) \to 1$

For $E = E_F$ (and $T > 0$ of course!), we get

$$f(E = E_F) = f(E_F) = \frac{1}{1 + e^0} = \frac{1}{2} = 0.5$$

This result is same as in case I. Thus value of fermi factor $f(E)$ is 0.5 at any temp. whether OK or any higher temperature.

With this discussion, we can see that shape of graph of $f(E)$ vs E at a given temp should be of the form

as shown in Fig. (2)

Fig. 2.
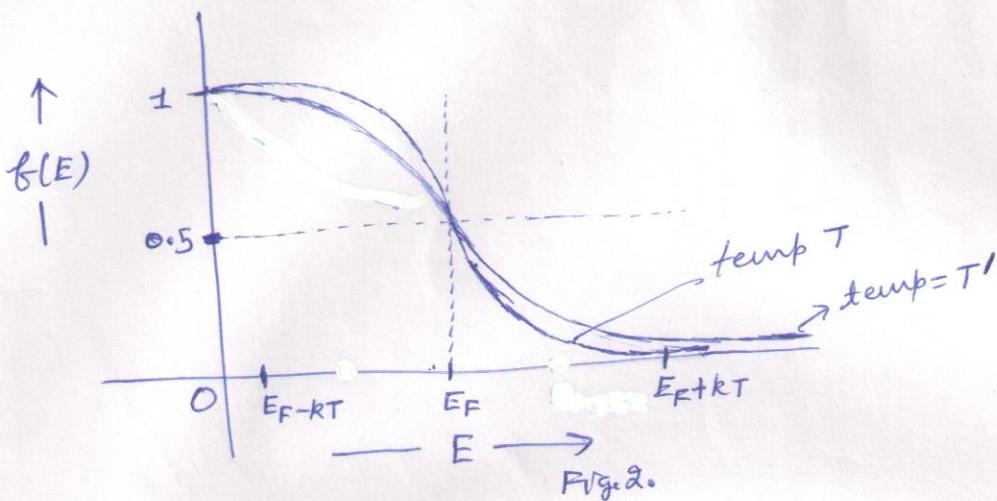
Let us now plot same graph at another temperature T' which is greater than T (i.e. $T' > T$). Consider the region $E > E_F$. Since

$$T' > T \quad \therefore \quad (E - E_F)/kT > (E - E_F)/kT'$$

$$\Rightarrow \quad f(E) \text{ at } T < f(E) \text{ at } T' \quad [\text{See equation ①}]$$

~~Now consider~~ Now consider region $E < E_F$. In this case we must remember that $E - E_F$ is negative. Therefore

$$(E - E_F)/kT < (E - E_F)/kT'$$

$$\therefore \quad f(E) \text{ at } T < f(E) \text{ at } T'$$

Thus graph of $f(E)$ at T' will be above graph of $f(E)$ at T in the region $E > E_F$ and it will be below graph of $f(E)$ at T in the region $(E < E_F)$, while $f(E) = \frac{1}{2}$ will be a crossover ~~point~~ point as shown in fig 2.

We shall make two changes in Fig. 2 for practical purposes.

(i) we will show energy along Y-axis and $f(E)$ along X-axis for practical purposes (although it appears to be somewhat illogical right now)

(ii) Actually tail of graph extends upto ∞, but for practical purposes

$f(E)$ does not change much in the deep regions, so we will show this graph touching the axes within a region of kT. With these changes, the variation of $f(E)$ at different temperatures (including $T = 0K$ case) is shown in fig. 3.
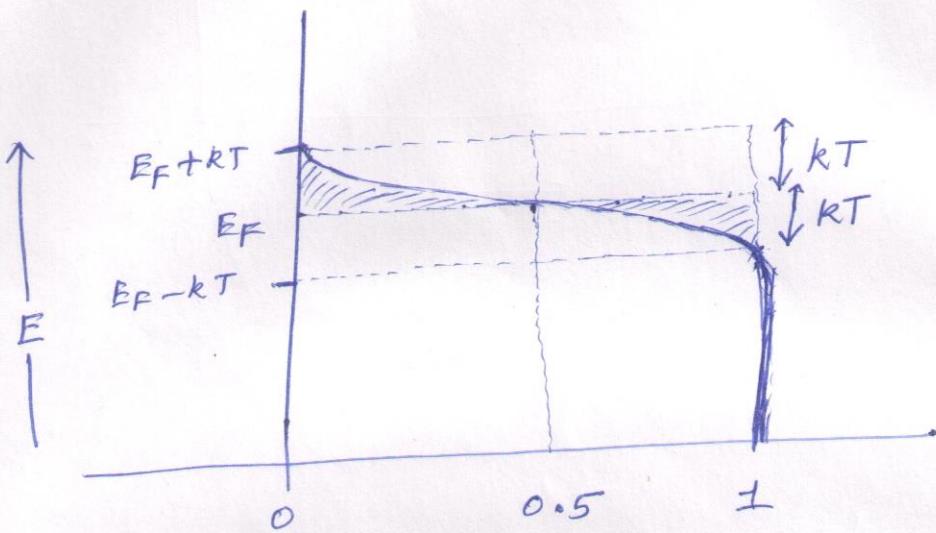

Fig. 3

Effective Mass (m^*) :- It is a general perception that mass of an electron in a solid is same as the mass of a free electron. However experimentally, it is observed that ~~in~~ inside some solids the mass of electron, when in motion, is more than the mass of a free electron, while for some other solids, it is less than mass of a free electron. We cannot associate this change in mass with relativistic effects due to two reasons.

(i) Speed of electron inside a solid is very small as compared to speed of light (Typically of the order of 10^5 m/s) so relativistic effects are supposed to be NIL.

(ii) Due to relativistic effects mass in motion is always more

while decrease in mass cannot take place. However experimentally decrease in mass is also observed in certain solids.

It is observed that when an electron moves in a solid, it experiences interactions from other electrons and positive ions present in its neighbourhood. The combination of electrons and nearby +ve ions form a potential U around the moving electron due to which

$$\left(\begin{array}{c}\text{graph in}\\\text{momentum}\\\text{space}\end{array}\right) \text{Fig. 1.} \left(\begin{array}{c}\text{graph in}\\\text{configuration}\\\text{space}\end{array}\right)$$

interaction takes place, which results in change in mass of electron moving inside a solid. Thus "Effective mass of an electron is the mass in the presence of lattice potential in a solid." It is denoted by symbol m^*. The expression for effective mass is given by

$$\boxed{m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}} \quad —①$$

Where $\hbar = \dfrac{h}{2\pi}$

$p = \hbar k = $ linear momentum

$E = $ Energy of electron moving in a solid.

The variation of energy of an electron with respect to propagation constant k in the conduction band and valence band is shown in Fig. 1. Equation ① can also be written as $\boxed{\dfrac{1}{m^*} = \dfrac{1}{\hbar^2}\dfrac{d^2E}{dk^2}} \quad —②$

Since $\dfrac{d^2E}{dk^2}$ represents curvature of graph between E & k. Thus from Fig 1 above, we can make following conclusions

(i) Near the bottom of band (V.B.) $m^* \simeq m$ (where m = mass of free electron) (because E and k graph is a parabola here, and for free electron potential

energy is zero, so that

$$E = KE + PE$$

$$= \frac{p^2}{2m} + 0$$

$$= \frac{\hbar^2 k^2}{2m}$$

(Note that for free electron use symbol m for mass and for bound electron use symbol m^* for mass)

$$\Rightarrow \quad E \propto k^2$$

which means that E & k graph of a free electron is a parabola.

(ii) At the points of inflexion A & A', ~~Eg~~ the graph between E and k changes between concave to convex shape so that $\frac{d^2E}{dk^2} = 0$ ∴ From ② $\frac{1}{m^*} = 0$ or $m^* = \infty$

This means that external potential of lattice cannot exert any action on the motion of electron in this region.

(iii) Between points 0 & A (or 0 & A') curvature of E and k curve is +ve so $\frac{1}{m^*}$ is +ve or effective mass of electron is +ve.

(iv) Beyond points A and A' (i.e. near the top of valence band), the E/k graph has negative curvature i.e. $\frac{d^2E}{dk^2}$ ⊗⊗ or m^* is negative. Thus effective mass is negative near this region.

The concept of effective mass provides satisfactory description of the charge carriers in crystals. In crystals like alkali metals, energy bands are partially filled and conduction takes place mainly due to electrons. However for crystals where energy band is nearly full, the vacancies having −ve charge & −ve mass are equivalent to particles of +ve charge and +ve mass, called holes.

Calculation of electron density in conduction band in an intrinsic semiconductor :-

Consider two narrowly spaced energy levels E and $E+dE$. Let number of electrons existing per unit volume of the sample in this energy range are dn. If density of energy states in this region is $Z(E)dE$ and fermi function is $f(E)$, then we have

$$dn = \left(\frac{\text{Energy states in the region } E \text{ \& } E+dE}{\text{Volume}}\right) \times \left(\begin{array}{c}\text{probability of}\\\text{a state to be}\\\text{filled by}\\\text{an electron}\end{array}\right)$$

$\qquad\qquad\downarrow$ density of states $\qquad\qquad\qquad\downarrow$ fermi function

$$= (Z(E)dE) \times f(E)$$

or $\quad dn = Z(E) f(E) dE \quad$ —①

The expression for density of states in energy region E \& $E+dE$ is given by :-

$$Z(E)dE = \cancel{\frac{4\pi}{h^3} (2m_e^*)} \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{\frac{1}{2}} dE \quad —②$$

Where m_e^* = effective mass of electron in ~~conduction~~ conduction band, E_c = energy corresponding to bottom edge of conduction band.

The expression for fermi function is given by

$$f(E) = \frac{1}{1+ e^{(E-E_F)/kT}} \approx e^{-(E-E_F)/kT} \quad —③$$

• (Neglecting 1 in comparison to exponential term)

For doing this article student has to remember following standard results :-

(i) The density of energy states in conduction band in small energy region dE around energy value E is given by

$$Z(E)dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{1/2} dE$$

(ii) $\displaystyle\int_0^\infty x^{\frac{1}{2}} e^{-ax} dx = \frac{\pi^{1/2}}{2a^{3/2}}$

(Well maths is sometimes challenging but always very exciting !)

This integration is challenge to ILATE concept learned in 10+2 classes

Put equations ② & ③ in ①, we get

$$dn = \left[\frac{4\pi}{h^3} (2m_e^*)^{3/2} (E-E_c)^{\frac{1}{2}} e^{-(E-E_F)/kT} \right] dE$$

$$= \left[\frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} (E-E_c)^{\frac{1}{2}} \times e^{-(E-E_c)/kT} \times e^{-(E_c-E_F)/kT} \right] dE$$

Total number of free electrons per unit volume in the conduct band are given by:

$$n = \int_{E_c}^{\infty} dn = \frac{4\pi}{h^3} (2m_e^*)^{\frac{3}{2}} e^{-(E_c-E_F)/kT} \int_{E_c}^{\infty} (E-E_c)^{\frac{1}{2}} e^{-(E-E_c)/kT} dE \qquad —④$$

In equation ④, we can put $a = \frac{1}{kT}$

and $E - E_c = x$ (say)

So that $dE = dx$

when $E = E_c$ then $x = E - E_c = E_c - E_c = 0$
when $E = \infty$ then $x = \infty - E_c = \infty$

∴ Integration part of equation ④ can be calculated as

$$\int_{E_c}^{\infty} (E-E_c)^{\frac{1}{2}} e^{-(E-E_c)/kT} dE = \int_{0}^{\infty} x^{\frac{1}{2}} e^{-ax} dx$$

$$= \frac{\sqrt{\pi}}{2 a^{3/2}}$$

$$= \frac{\sqrt{\pi} \times (kT)^{3/2}}{2}$$

Put this value in ④, we get

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{-(E_c-E_F)/kT} \times \frac{\sqrt{\pi} (kT)^{3/2}}{2}$$

$$\Rightarrow \boxed{n = 2\left(\frac{2m_e^* \pi kT}{h^2}\right)^{3/2} e^{-(E_c-E_F)/kT}} \quad —— ⑤$$

or $$\boxed{n = N_c \, e^{-(E_c-E_F)/kT}} \quad —— ⑥$$

where $$\boxed{N_c = 2\left(\frac{2m_e^* \pi kT}{h^2}\right)} \quad —— ⑦$$

N_c is called effective density of (energy) states in the conduction band. Equation ⑤ shows that population density of electrons (n) in the conduction band is a function of temperature and energy of fermi level.

Calculation of hole density (p) in the valence band (VB) of an intrinsic Semiconductor :-

We know that fermi function $f(E) = \dfrac{1}{1 + e^{-(E-E_F)/kT}}$

represents the probability of finding an electron in the energy state E. However a hole is created when an electron is taken out of a bond/band. This ~~means~~ means that presence of hole is equivalent to absence of electron. ∴ If Prob(h) is probability of a hole, then we can write

$$Prob(h) = 1 - \text{Probability of electron} = 1 - f(E)$$

$$\therefore \text{Prob}(h) = 1 - \frac{1}{1 + e^{(E-E_F)/kT}}$$

$$= \frac{\cancel{1} + e^{(E-E_F)/kT} - \cancel{1}}{1 + e^{(E-E_F)/kT}}$$

$$= \frac{e^{(E-E_F)/kT}}{1 + e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT} + 1}$$

$$= \frac{1}{e^{(E_F-E)/kT} + 1}$$

$$\approx \frac{1}{e^{(E_F-E)/kT}}$$

$$= e^{-(E_F-E)/kT} \quad —— \text{①}$$

$\left(\begin{array}{l} \therefore \text{For a state in Valence band } E \ll E_F \\ \text{so 1 can be neglected} \\ \text{in comparison to exponential} \\ \text{part} \end{array} \right)$

The density of energy states within the energy range E and E+dE of valence band is given by:

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_v - E)^{1/2} dE \quad —— \text{②}$$

(where m_h^* = effective mass of hole and E_v is energy corresponding to top edge of Valence band)

Let dp = concentration of holes per unit volume (also called as hole density) in the energy range E to E+dE within Valence band

$$\therefore dp = \text{Prob}(h) \times Z(E) dE$$

$$= e^{-(E_F - E)/kT} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{\frac{1}{2}} dE$$

$$= e^{-(E_F - E_V)/kT} \times e^{-(E_V - E)/kT} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{\frac{1}{2}} dE \quad \text{——③}$$

Total number of holes per unit volume in the valence band are given by:

$$p = \int_{-\infty}^{E_V} dp$$

$$= e^{-(E_F - E_V)/kT} \times \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{\frac{1}{2}} e^{-(E_V - E)/kT} dE \quad \text{——④}$$

If we put $E_V - E = x$, $\frac{1}{kT} = a$ (say), then

we get $0 - dE = dx$ or $dE = -dx$, when $E = -\infty$ then

$x = E_V - E_\bullet = E_V - (-\infty) = +\infty$, when $E = E_V$, then $x = 0$

∴ Integration part in ④ can be solved as follows

$$\int_{-\infty}^{E_V} (E_V - E)^{\frac{1}{2}} e^{-(E_V - E)/kT} dE = \int_{\infty}^{0} x^{\frac{1}{2}} e^{-ax} (-dx) = \int_{0}^{\infty} x^{\frac{1}{2}} e^{-ax} dx$$

$$= \frac{\sqrt{\pi}}{2 a^{3/2}} = \frac{\sqrt{\pi} (kT)^{3/2}}{2} \quad \text{——⑤}$$

∴ Equation ④ becomes

$$p = e^{-(E_F - E_V)/kT} \times \frac{4\pi}{h^3}^{2} (2m_h^*)^{3/2} \times \frac{\sqrt{\pi} (kT)^{3/2}}{2}$$

or

$$\boxed{p = 2 \left(\frac{2 m_h^* \pi kT}{h^2} \right) e^{-(E_V - E_F)/kT}} \quad \text{——⑥}$$

or

$$p = N_V e^{-(E_F - E_V)/kT} \quad —— \quad (7)$$

where

$$N_V = 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2} \quad —— \quad (8)$$

N_V is called Effective density of States in the ~~silicon~~ Valence band. Equation ⑦ shows that hole density/concentration in Valence band is also a function of fermi level E_F and temperature T. It should be noted that N_c in previous article and N_V in the present discussion also depend on temperature T. So to understand dependence of n & p on T these factors (N_c & N_V) cannot be ignored and complete understanding can come only by plotting n & p with temperature.
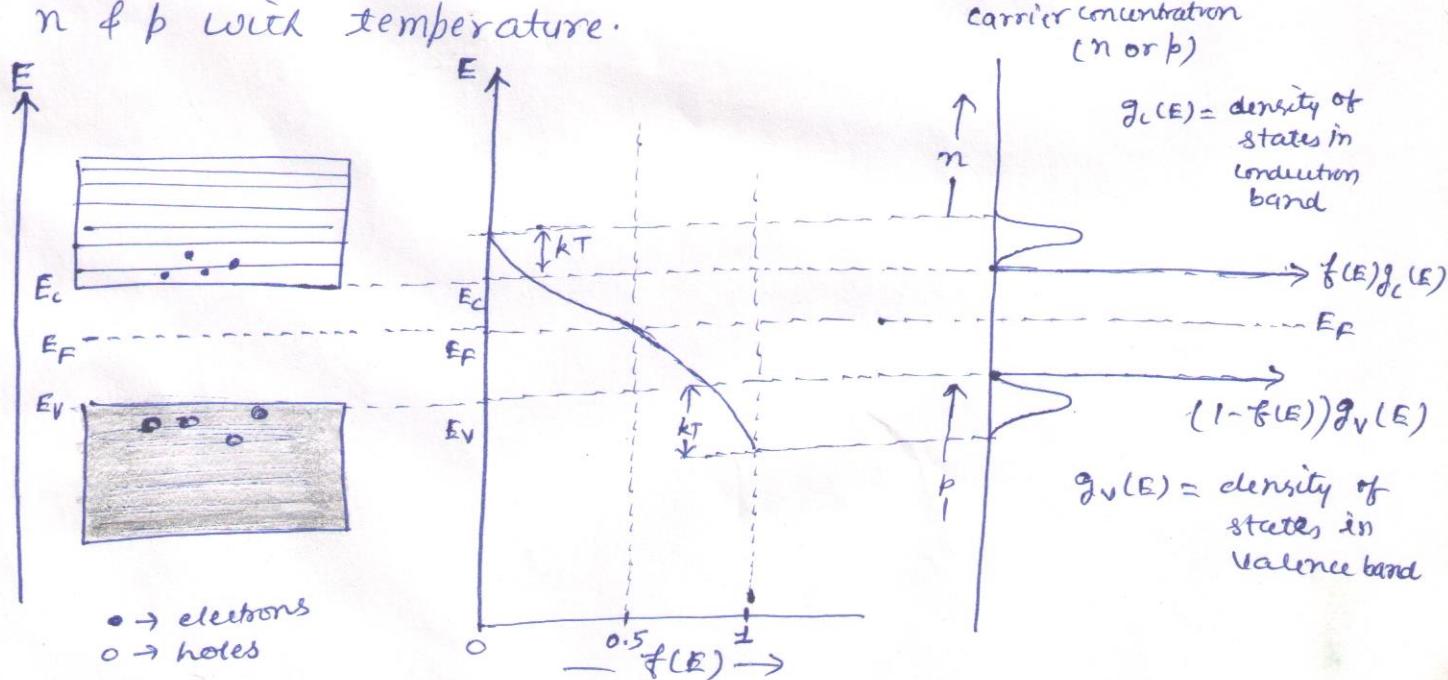
Fig 1.

The variation of carrier concentration within ~~the~~ conduction and valence band at a given temperature T is shown in figure 1. It is clear that at a given temp. T, electrons are mostly localized in region kT above E_c and holes are localized in small region ~~be~~ kT below E_V.

Intrinsic carrier concentration and its dependence with temperature:-

Whenever a single bond breaks in a semiconductor, then and electron-hole pair (EHP) is generated. Thus at any temperature T, the number of free electrons or holes per unit volume of the sample will be equal. Because of equal value of carrier concentration, we denote it by a general symbol n_i and call it as intrinsic density or intrinsic concentration. Thus if n is concentration of free electrons and p is concentration of holes, then we can write

$$n_i = n \qquad \text{①}$$
$$\text{and } n_i = p \qquad \text{②}$$

Multiply ① & ②, we get $n_i^2 = np \qquad \text{③}$

However, expressions of n & p are given as follows:

$$n = N_c \, e^{-(E_c - E_F)/kT} \qquad \text{④}$$
$$\text{and } p = N_v \, e^{-(E_F - E_v)/kT} \qquad \text{⑤}$$

Put values from ④ and ⑤ in ③, we get

$$n_i^2 = N_c N_v \, e^{-(E_c - E_v)/kT}$$
$$= N_c N_v \, e^{-E_g/kT} \qquad \text{⑥}$$

where $E_g = E_c - E_v \qquad \text{⑦}$

E_g represents separation between lowest energy level of conduction band and highest energy level of valence band. It is called band gap energy or simply as band gap.

The values of N_c and N_v are given by following expressions:

$$N_c = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} \times (m_e^*)^{3/2} \qquad \text{⑧}$$

$$\text{and } N_v = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} \times (m_h^*)^{3/2} \qquad \text{⑨}$$

Put values from ⑧ & ⑨ in ⑥, we get

$$n_i^2 = 4 \left(\frac{2\pi kT}{h^2} \right)^{3} (m_e^* m_h^*)^{3/2} e^{-E_g/kT} \qquad \text{⑩}$$

$$\text{or } \boxed{n_i = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT}}$$

To understand the dependence of n_i on temperature T, we can rewrite equation ⑩ as follows:

$$n_i = 2\left(\frac{2\pi k}{h^2}\right)^{3/2} (m_e^* \, m_h^*)^{3/4} \; T^{3/2} \; e^{-Eg/2kT} \qquad ⑪$$

$$\therefore \quad n_i \; \propto \; T^{3/2} e^{-Eg/2kT} \qquad\qquad ⑫$$

From ⑫, we can draw following conclusions:-

(i) Intrinsic carrier concentration is independent of the energy of fermi level (EF)

(ii) The intrinsic carrier concentration has exponential dependence on band gap energy (Eg) between valence and conduction band.

(iii) Intrinsic carrier conc. is strongly dependent on temperature.

Since $e^{-(Eg/2kT)} = 1 - \frac{Eg}{2kT} + \frac{Eg^2}{2!\,(4k^2T^2)} - \frac{Eg^3}{3!\,(8k^3T^3)} + \cdots$

$\therefore$ at small temperatures exponential term dominates while at large temperature $(T \to \infty)$ this exponential factor approaches to unit (i.e. as $T \to \infty$ $e^{-Eg/2kT} \to 1$)

$\therefore$ at low temperatures we can assume $n_i \propto e^{-(Eg/2kT)}$ (from ⑫)

and at high temperatures we can assume $n_i \propto T^{3/2}$

(iv) Presence of factor 2 in ⑫ indicates that two charge carriers (one electron and one hole) are produced when a covalent bond is broken. This process is called production of electron-hole pair (EHP). Thus concentration of electrons in conduction band and concentration of holes in valence band are always equal in an intrinsic semiconductor.

Expression for energy ~~is~~ of fermi level of an intrinsic $\overset{SC}{}$ Semiconductor : We know that concentration of electrons in conduction band of an intrinsic semiconductor is given as :-

$$n = N_c \, e^{-(E_c - E_F)/kT} \quad\quad \text{①}$$

and ~~is~~ concentration of holes in the valence band is given as

$$p = N_v \, e^{-(E_F - E_v)/kT} \quad\quad \text{②}$$

Where

$$N_c = 2 \left(\frac{2 m_e^* \pi kT}{h^2} \right)^{3/2} \quad\quad \text{③}$$

and

$$N_v = 2 \left(\frac{2 m_h^* \pi kT}{h^2} \right)^{3/2} \quad\quad \text{④}$$

Since conc. of electrons and holes for intrinsic semiconductor are equal, therefore, we can put

$$n = p$$

$$\Rightarrow \quad N_c \, e^{-(E_c - E_F)/kT} = N_v \, e^{-(E_F - E_v)/kT}$$

Taking natural log on both sides, we get-

$$\ln(N_c) - \left(\frac{E_c - E_F}{kT} \right) = \ln(N_v) - \left(\frac{E_F - E_v}{kT} \right) \quad (\because \ln(e) = 1)$$

$$\Rightarrow \ln(N_c) - \ln(N_v) = \left(\frac{E_c - E_F}{kT} \right) - \left(\frac{E_F - E_v}{kT} \right)$$

$$\Rightarrow \ln\left(\frac{N_c}{N_v} \right) = \frac{E_c + E_v - 2E_F}{kT}$$

$$\Rightarrow kT \ln\left(\frac{N_c}{N_v}\right) = E_c + E_v - 2E_F$$

$$\Rightarrow 2E_F = E_c + E_v - kT \ln\left(\frac{m_e^*}{m_h^*}\right)^{3/2} \quad (\text{using } ③ \text{ \& } ④)$$

$$\boxed{E_F = \frac{E_c + E_v}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)} \quad —⑤$$

Fig.
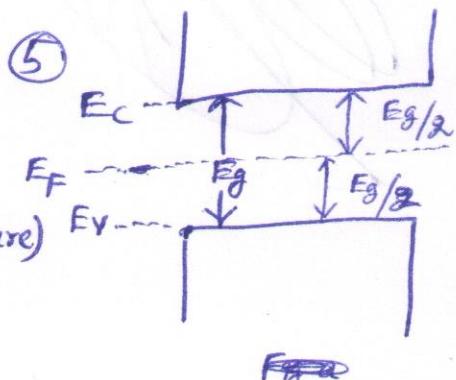
Let E_g = band gap energy (as shown in figure here)

∴ From fig., we can write that

$$E_g = E_c - E_v \quad\quad\quad\quad —⑥$$

$$\Rightarrow 2E_v + E_g = E_c - E_v + 2E_v$$

or $\quad 2E_v + E_g = E_c + E_v \quad$ Putin ⑤, we get

$$E_F = \frac{2E_v + E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)$$

or $\quad \boxed{E_F = E_v + \frac{E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)} \quad —⑦$

If we assume that effective mass of electron and hole are equal (i.e. $m_e^* = m_h^*$), then equation ⑦ becomes

$$\boxed{E_F \simeq \frac{E_v + E_g}{2}} \quad —⑧$$

or

$$\boxed{E_F \simeq E_c - \frac{E_g}{2}} \quad —⑨ \quad\quad (\because E_v = E_c - E_g)$$

From ⑧ & ⑨, we see that E_F as more than E_v by a factor of $\frac{E_g}{2}$ and it is less than E_c by same factor $\frac{E_g}{2}$. This means that

Fermi level for an intrinsic semiconductor lies in the middle of energy gap

*Note :- Some times energy of valence band top edge is taken as a reference (i.e. $E_v = 0$). In that case we can use the relation

$$E_F = \frac{Eg}{2}$$

Also note that for intrinsic semiconductor, fermi level is not a real level as it lies in middle of energy gap, where no energy state exists.

Variation of Fermi level with temperature in an intrinsic Semiconductor :- The expression for Fermi energy in an intrinsic semiconductor is given by:

$$E_F = E_v + \frac{Eg}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right) \underline{\hspace{1cm}} \textcircled{1}$$

Case I   If $m_e^* = m_h^*$

then ① becomes $E_F = E_v + \frac{Eg}{2}$

∴ E_F is independent of temperature
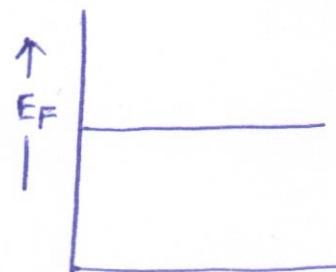
The graph between E_F & T is shown in fig(i)

Fig(i)

Case II   If $m_e^* > m_h^*$

∴ $\ln\left(\frac{m_e^*}{m_h^*}\right)$ is positive. Hence from equation ① we see that E_F decreases linearly with temperature as shown in Fig(ii).
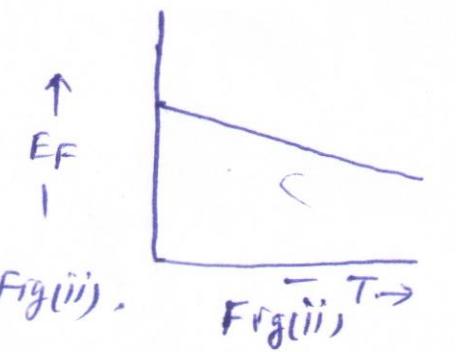

Fig(ii)

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Case III   If $m_e^* < m_h^*$

In this case $\ln\left(\dfrac{m_e^*}{m_h^*}\right)$ is negative

∴ From ① we can conclude that

E_F increases linearly with temperature

as shown in Fig(iii).

fig(iii)
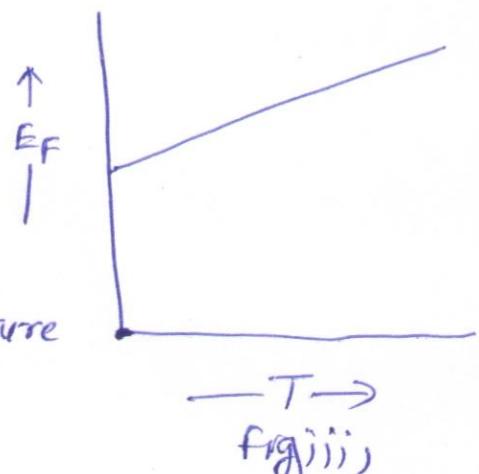
Expression for conductivity of an intrinsic Semiconductor and its variation with temperature:-

We know that current in a conductor is related to the drift velocity according to the relation

$$I = nAe\,v_d \quad\text{——} \quad ①$$

where   n = concentration of free electrons

A = Area of cross section of metallic conductor

e = ~~mag~~ charge on electron

v_d = drift velocity of electrons

In case of Semiconductors, both electrons and holes contribute in conduction. Let I_e and I_h are conventional current due to electron and hole in a semiconductor material respectively.

Let   A = area of cross section of Semiconductor

l = length of Semiconductor

e = charge on electron/hole (magnitude only)

μ_e = electron mobility

μ_h = hole mobility

V = applied potential difference

R = resistance of Semiconductor

ρ = resistivity of Semiconductor
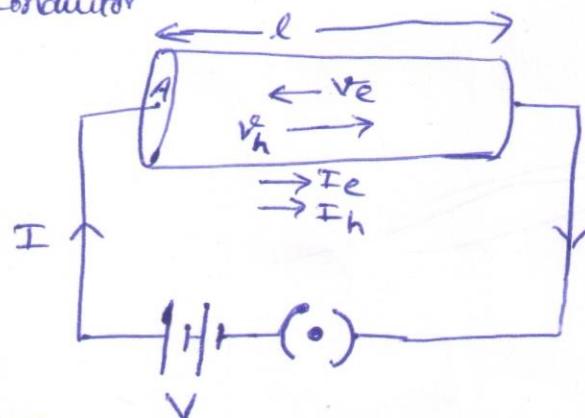
σ = conductivity of semiconductor

E = applied electric field (by battery)

n = concentration of electrons (free electrons only)

p = concentration of holes

$\therefore$ Using ①, we can write

$$I_e = nAe\,v_e \quad —②$$

and $\qquad I_h = pAe\,v_h \quad —③$

Let I = total conventional current flowing through the semiconductor

$\therefore \quad I = I_e + I_h \quad \left(\because \text{both conventional currents are in same}\atop \text{direction}\right)$

$$= Ae\,(nv_e + pv_h) \quad —④$$

But electron & hole mobilities are given by following expressions

$$\mu_e = \frac{v_e}{E} \quad ; \quad \mu_h = \frac{v_h}{E}$$

$\therefore \quad v_e = \mu_e E = \mu_e \dfrac{V}{\ell} \qquad \left(\because E = \dfrac{V}{\ell}\right)$

lly $v_h = \dfrac{\mu_h V}{\ell}$

Put in ④ $\therefore \quad I = \dfrac{V}{\ell} Ae\,(n\mu_e + p\mu_h)$

$\therefore \quad \cancel{I} = \dfrac{\cancel{I}R}{\ell} Ae\,(n\mu_e + p\mu_h) \quad \left(\because V = IR\right)$

$\Rightarrow \quad 1 = \dfrac{\rho\cancel{\ell}}{A} \times \dfrac{Ae}{\cancel{\ell}}\,(n\mu_e + p\mu_h) \quad \left(\because R = \dfrac{\rho\ell}{A}\right)$

$\Rightarrow \quad \dfrac{1}{\rho} = e\,(n\mu_e + p\mu_h)$

or $\qquad \boxed{\sigma = e\,(n\mu_e + p\mu_h)} \quad —⑤ \quad \left(\because \sigma = \dfrac{1}{\rho}\right)$

Equation ⑤ gives expression for conductivity of an intrinsic

Semiconductor in terms of mobilities.

But for an intrinsic Semiconductor

$$n = p = n_{i0} = \sqrt{N_c N_v} \; e^{-Eg/2kT} \qquad —6$$

∴ Equation ⑤ becomes   (See page ⑭, equation no. ⑥)

$$\sigma = e(\mu_e + \mu_h)\sqrt{N_c N_v} \; e^{-Eg/2kT} \qquad —7$$

or

$$\boxed{\sigma = \sigma_0 \; e^{-Eg/2kT}} \qquad —8$$

Where $\boxed{\sigma_0 = e(\mu_e + \mu_h)\sqrt{N_c N_v}} \qquad —9$

σ_0 is some constant

From ⑧, we see that conductivity
of an intrinsic Semiconductor increases
exponentially with temperature as shown
in fig(i) below. Moreover for $T=0, \sigma = 0$
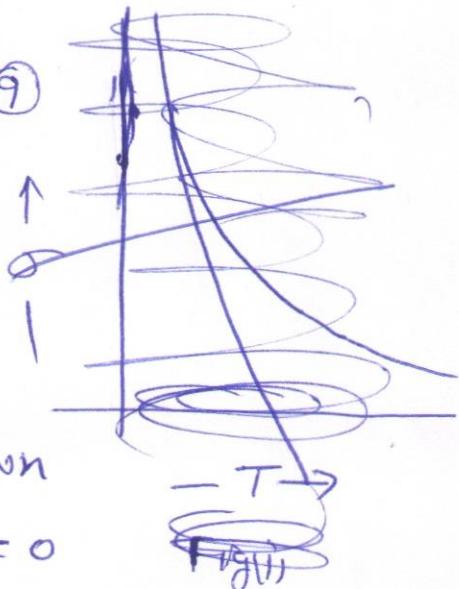
∴ at absolute 0K, a Semiconductor behaves like perfect insulator.
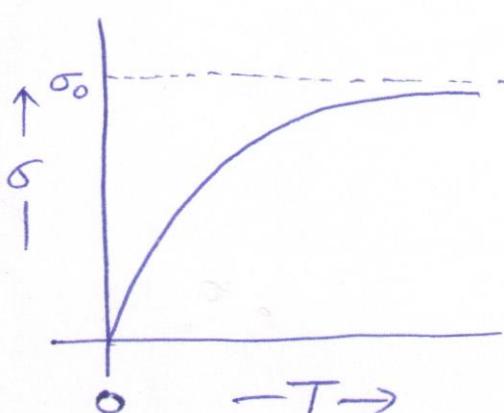
(Fig (i))
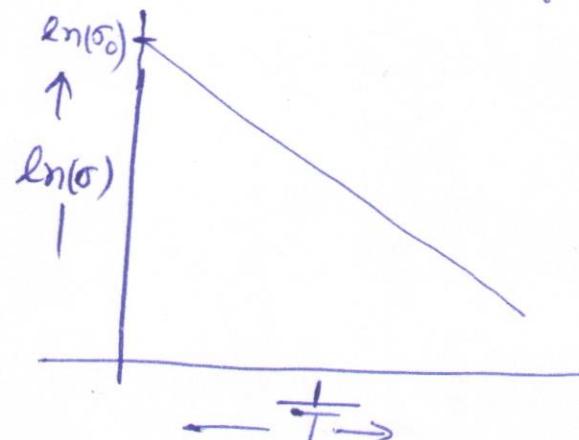
(Fig (ii))

As T is increased σ increases and at $T=\infty$, $\sigma \to \sigma_0$. Thus
Constant σ_0 is maximum conductivity of intrinsic semiconductor,

which it achieves at ∞ Kelvin.

If we take natural logarithm on both sides of ⑧, we get

$$\ln(\sigma) = \ln(\sigma_0) - \frac{E_g}{2kT} \quad —⑩$$

Equation ⑩ shows that graph between $\ln(\sigma)$ and $\frac{1}{T}$ is a straight line. ~~The slope of~~ This variation is shown in fig(ii) above. This relation is very important as it can be used to find the band gap energy of an intrinsic Semiconductor. A graph is plotted between $\ln(\sigma)$ & $\frac{1}{T}$ by experimentally measuring conductivity at different values of temperature. Then slope of graph gives $\frac{E_g}{2k}$ value, so that

$$E_g = 2k \times \text{slope of graph}$$

Further   Intercept $= \ln(\sigma_0)$

$$\therefore \quad \sigma_0 = e^{\text{intercept}}$$

Since conductivity measurement is relatively difficult than resistance measurement (which is just equal to ratio of instantaneous voltage to current), so E_g is usually measured from plot of logarithm of resistance. ⑧ can be written as

$$\frac{1}{S} = \frac{1}{S_0} e^{-E_g/kT}$$

$$\Rightarrow \quad \boxed{S = S_0 \, e^{E_g/kT}} \quad —⑪$$

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If we put $S = S_0$ in ⑪, we get $T = \infty$

Thus S_0 is resistivity of semiconductor at ∞ temperature.

Since $R = \dfrac{S\ell}{A}$ ⟹ $S = \dfrac{RA}{\ell}$ put in ⑪, we get

$$\dfrac{RA}{\ell} = S_0\, e^{Eg/2kT} \Rightarrow R = \dfrac{S_0\ell}{A}\, e^{Eg/2kT}$$

or $\boxed{R = R_0\, e^{Eg/2kT}}$

Where $R_0 = \dfrac{S_0\ell}{A}$

$\therefore \boxed{\ln(R) = \ln(R_0) + \dfrac{Eg}{2kT}}$

$\therefore \boxed{Eg = 2k \times \text{slope}}$

$\ln(R)$ ↑

$-\dfrac{1}{T} \rightarrow$

O
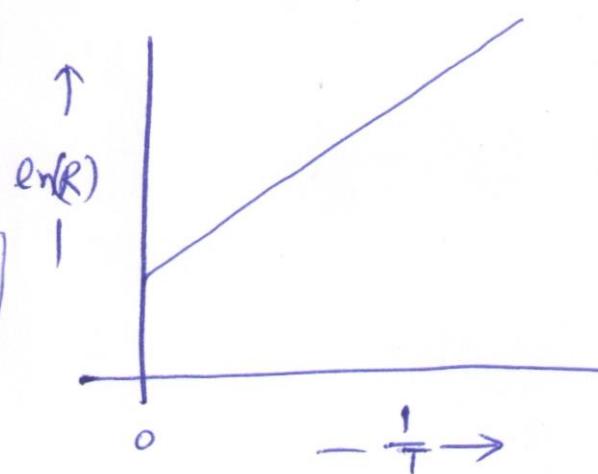
Energy band diagram of n-type extrinsic semiconductor and Variation of carrier concentration with temperature:

We know that an n-type extrinsic semiconductor is obtained by adding a pentavalent impurity in semiconductor sample.

(a)

$O \longrightarrow Si$
● $\longrightarrow P$

(b)

E_C, E_D, E_V, Eg, CB, VB

conc. = n
conc. = N_D
conc. = N_D^+
conc. = p

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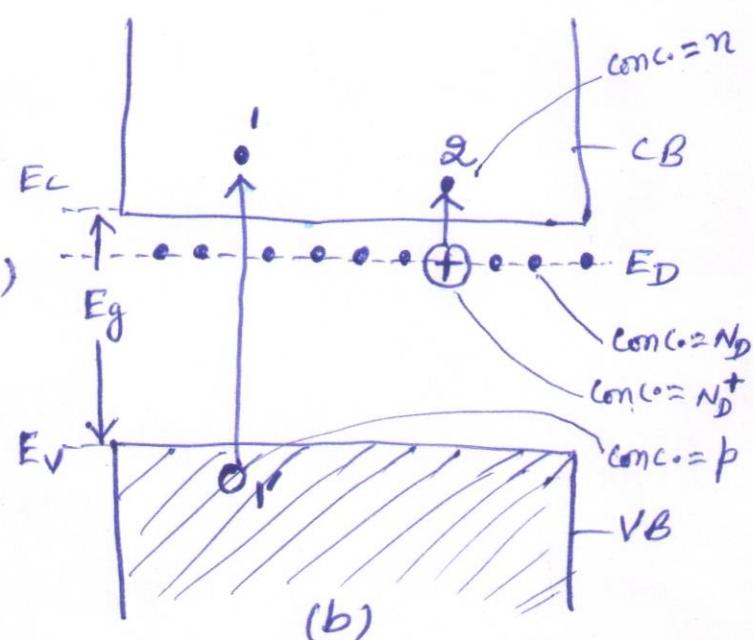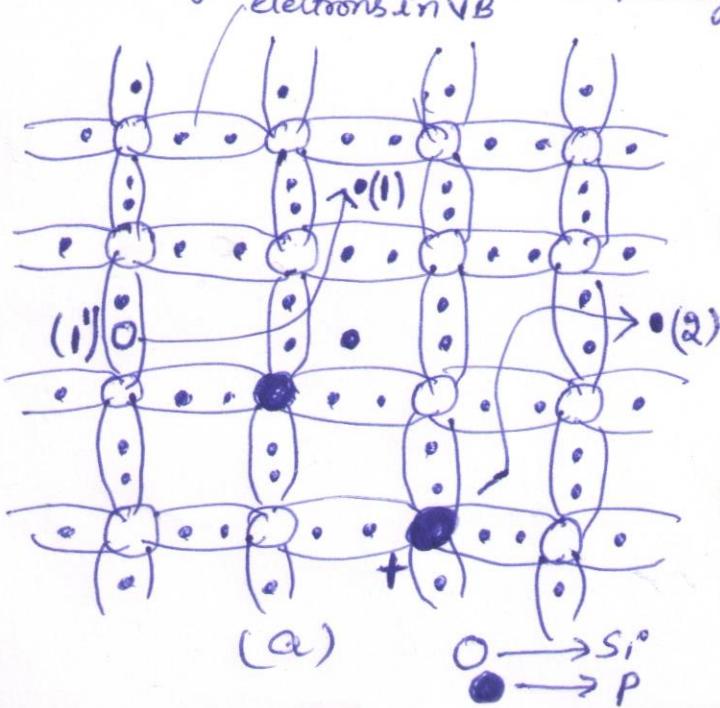
To understand the difference in conductivity behaviour of n-type semiconductor and an intrinsic semiconductor, let us assume that pentavalent impurity atoms $_{15}P$ are added into $_{14}Si$ semiconductor sample. For $_{14}Si° = 2, 8, 4$, there are four electrons in outermost shell and for $15P = 2, 8, 5$, there are 5 electrons in outmost shells. The electronic/crystal structure has been shown in 2-D plane in fig (a) above. We observe that all Si-Si↑ bonds are saturated and each Si° atom forms 4 covalent
covalent bonds with↑ nearest ~~other~~ Si atoms. All these electrons,
other
which are involved in covalent bond formation are very tightly bound, therefore, cannot take part in conduction. All such electrons form Valence Band (VB), which is shown as shaded region in the band structure (fig(b)). When temp. is absolute zero, then no electron in semiconductor is free, therefore, conduction band is completely empty.

On increasing temperature an electron (like electron (1)) in the covalent bond can come out in the crystal and becomes free. Corresponding to this a +ve vacancy called hole (like hole number (1')) is produced in covalent bond. This transition is shown by vertical arrow $1' \longrightarrow 1$ in band structure and process is called electron-hole pair (EHP) production. This type of process dominates during conductivity of intrinsic semiconductor and $n = p$ in such cases.

However if we consider impurity atom $_{15}P$ in the

crystal sturcture (shown as shaded atom), then out of 5 outermost electrons, four are utilized in making covalent bonds with nearest Si atoms and 5th electron cannot do that. This fifth electron is still attached to $_{15}P$ nucleus by relatively less attraction, than other 4 electrons used in forming covalent bonds. This 5th electron is therefore more unstable than other tightly bound electrons and less unstable than completely free electrons (like electron (1)), which lie in Conduction band. Thus it is clear that energy of this 5th weakly bound nucleus will be slightly less than that of electrons in CB (i.e. free electrons). Same explannation is applicable for all such 5th electrons of $15P$ impurity atoms. Thus these 5th $\bar{e}s$ of impurity atoms create a real filled energy level of energy E_D just below the bottom edge of CB. This level, which is created in the energy gap region just below E_C due to addition of Pentavalent impurity is called Donor level (shown in fig.(b)).

The difference $E_C - E_D$ is Very small as compared to band gap energy E_g and ~~it can be easily~~

such small energy $(E_C - E_D)$ can be easily provided by slightly increasing temperature of n-type Semiconductor. When, it is done, then an electron from donor level will ~~then~~ undergo
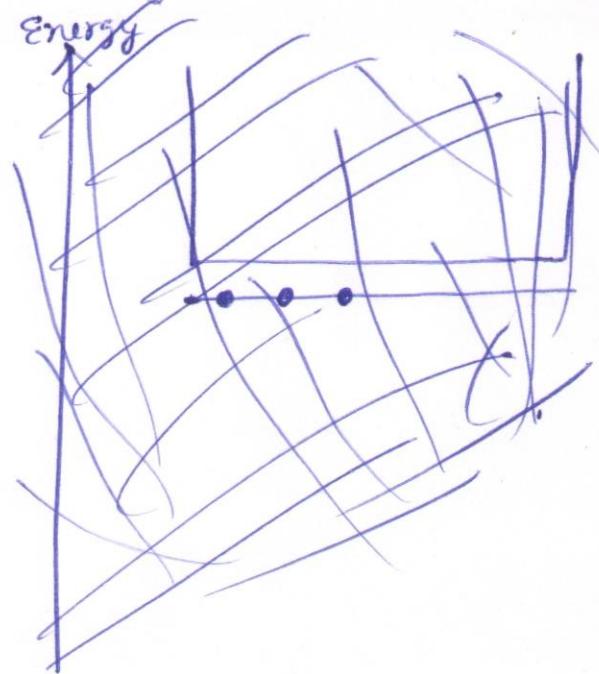
<u>SC</u>

transition to CB (like electron (2) in fig. (a) & (b)). Since this electron was not a part of any covalent bond before transition. Therefore, this transition cannot create any hole in VB (see fig(a) for better understanding), rather it will create an ~~immobile~~ immobile +ve ion P^+ in the donor level. With this explannation it is clear that for an n-type semiconductor number of free electrons in CB are always more than number of holes in VB (ie. $n >> p$). Thus electrons are majority carriers and holes are minority carriers in an n-type semiconductor.

fig ⓒ

(C₃)
$N_D^+ = N_D$
$n >> N_D$
$n >> N_D^+$
$n \approx p$

(C₂)
$n \approx N_D$
$N_D^+ \approx N_D$
$p \approx 0$

(C₁)
$n \approx N_D^+$
$N_D^+ < N_D$
$p \approx 0$
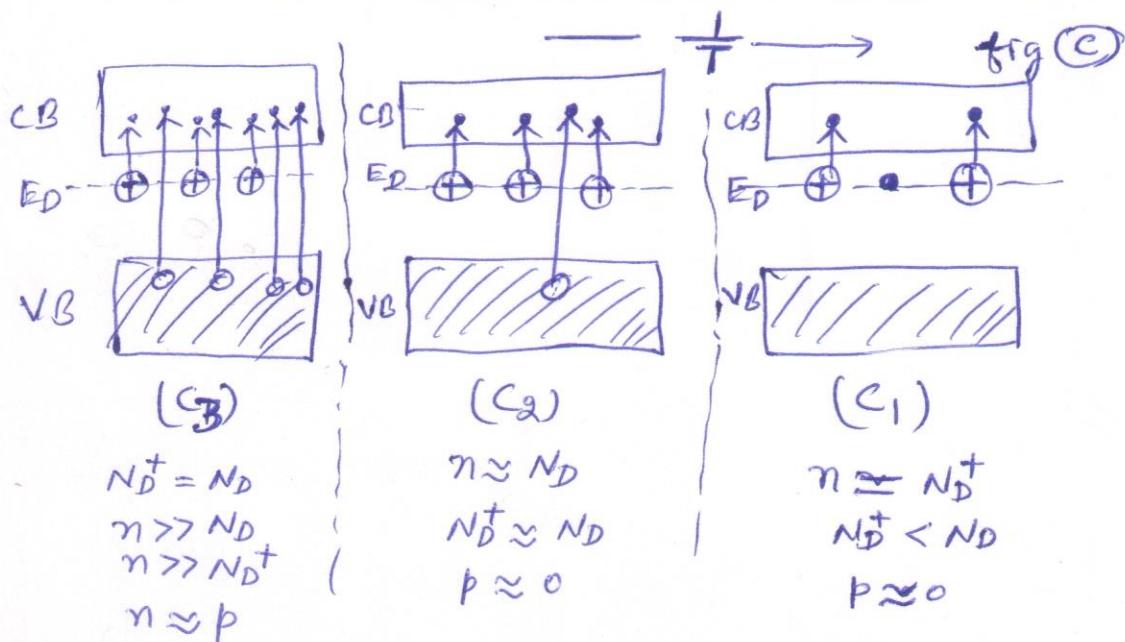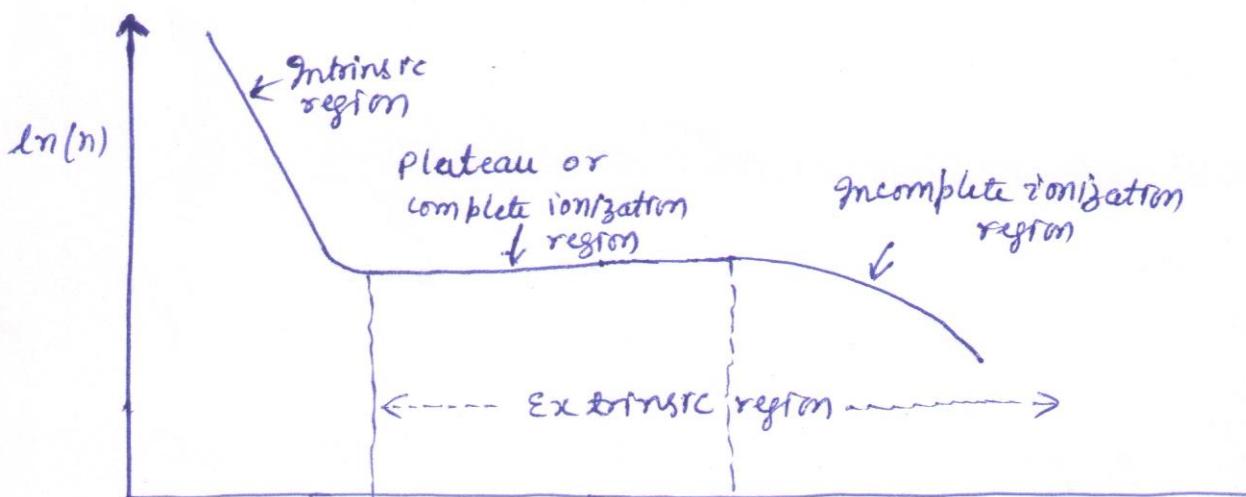
The general dependence of electron concentration on temperature for n-type semiconductor is shown in figure (c) above. At 0 Kelvin $\left(\frac{1}{T}=\infty\right)$, donor atoms are not ionized, therefore no conduction electron is available and material behaves like perfect insulator. When temp. is slightly increased then donor atoms start ionizing and electrons from E_D start reaching in CB. For each such transition from E_D to CB an electron is created in CB and an ion in E_D. The region is called partial ionization region. In this region conductivity of n-type semiconductor is purely due to electrons in CB and no hole is generated in VB. Moreover as temperature is increased ($\frac{1}{T}$ decreases), conductivity rises sharply.

Let $n =$ conc. of electrons in CB

$N_D^+ =$ conc. of ion generated in Donor level (E_D)

$N_D =$ conc. of donor atoms in Donor level

$p =$ conc. of holes in VB

∴ at low temp. (partial ionization region) we have

$$n = N_D^+, \qquad N_D^+ < N_D, \quad p = 0$$

This region is shown by region C_1 in fig (c)

If we continue to increase temp., a stage is reached where all donor atoms get ionized. With further increase in temp. electron conc. does not rise. This region (C_2) is called plateu or complete ionization region. It is also called Depletion region. In this region carrier conc or conductivity is almost independent of temperature. Moreover $n \approx N_D$

$$N_D^+ = N_D$$
$$p \approx 0$$

Both regions C_1 & C_2 combinedly constitute extrinsic region.

At very high temperature $\left[\left(\frac{1}{T} \to 0\right), (region\ (C_3))\right]$ electrons from VB also start transfering to CB and corresponding to each such transfer a hole is generated in VB and this region is called intrinsic behaviour region and EHPs are generated in this region. Obviously $n >> N_D$ and $n \approx p = n_i$ and at such very high temperatures material behaves like an intrinsic semiconductor.

Expression for carrier concentration in n-type semiconductor:

Consider an n-type semiconductor in the incomplete ionization region. Let $n =$ conc. of electrons in CB

$N_D^+ =$ conc. of ions in Donor level (E_D)

$N_D =$ total conc. of donor atoms

$N_D^o =$ conc. of unionized donor atoms

Since number of holes in VB are negligibly small, therefore, carrier conc. is just approximated by electron conc. in CB. Moreover, too incomplete ionization region, we can write

$$n = N_D^+$$

$$= N_D - N_D^o$$

$$= N_D - N_D f(E_D)$$

$$= N_D (1 - f(E_D))$$

$$= N_D \left[1 - \frac{1}{1 + e^{(E_D - E_F)/kT}}\right]$$

$$= N_D \left[\frac{e^{(E_D - E_F)/kT}}{1 + e^{(E_D - E_F)/kT}}\right]$$

where $f(E_D) =$ Fermi function corresponding to energy of donor level E_D at temp. T

$$= N_D \left[\frac{\overset{SC}{1}}{\frac{1}{e^{(E_D-E_F)/kT}} + 1} \right]$$

$$= N_D \left[\frac{1}{e^{-(E_D-E_F)/kT} + 1} \right] \underline{\qquad} ①$$

However E_F is a few kT times above E_D $\therefore e^{-(E_D-E_F)/kT} >> 1$

Hence eq. ① can be approximated as

$$n \simeq N_D\, e^{(E_D-E_F)/kT} \underline{\qquad} ②$$

But the electron conc. in CB is given by

$$n = N_C\, e^{-(E_C-E_F)/kT} \underline{\qquad} ③$$

comparing ① & ②, we get

$$N_D\, e^{(E_D-E_F)/kT} = N_C\, e^{-(E_C-E_F)/kT}$$

Take natural log on both sides, we get

$$\ln N_D + \frac{(E_D - E_F)}{kT} = \ln N_C - \frac{(E_C - E_F)}{kT}$$

$$\therefore \ln\left(\frac{N_C}{N_D}\right) = \frac{E_D - E_F + E_C - E_F}{kT}$$

$$\therefore 2E_F = E_D + E_C - kT \ln\left(\frac{N_C}{N_D}\right)$$

$$\Rightarrow E_F = \frac{E_D + E_C}{2} + \frac{kT}{2} \ln\left[\frac{N_D}{N_C}\right]$$