Subject: Mathematics I MST - I Subject code: BSC-103

Solution + Marking Scheme.

Q1. (a) Solve $(4xy + 3y^2 - x)dx + x(x+2y)\,dy = 0$ —(1)

Compare with $M(x,y)dx + N(x,y)\,dy = 0$

$M = 4xy + 3y^2 - x$ $N = x(x+2y)$

$\frac{\partial M}{\partial y} = 4x + 6y$, $\frac{\partial N}{\partial x} = 2x + 2y$ (1)

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Eqn (1) is not exact

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{4x + 6y - 2x - 2y}{x^2 + 2xy} = \frac{2}{x} = f(x)$$

$$I.F = e^{\int f(x)\,dx} = e^{2\log_e x} = x^2$$ (1½)

Multiply eqn. (1) by x^2

$$(4x^3y + 3x^2y^2 - x^3)\,dx + (x^4 + 2x^3y)\,dy = 0$$

Soln is

$$\int_{y\,\text{const}} M\,dx + \int N(\text{not containing } x)\,dy = c$$ (1½)

$$\int(4x^3y + 3x^2y^2 - x^3)\,dx + \int 0\,dy = c$$

$$x^4y + x^3y^2 - \frac{x^4}{4} = c$$

Scanned with CamScanner

(b) $4y^2p^2 + 2pxy(3x+1) + 3x^3 = 0$

$4y^2p^2 + 6px^2y + 2pxy + 3x^3 = 0$

$2yp(2yp + x) + 3x^2(2py + x) = 0$ 1

$(2yp + x)(2yp + 3x^2) = 0$

$2yp + x = 0$ and $2yp + 3x^2 = 0$

$p = \frac{-x}{2y}$, $p = \frac{-3x^2}{2y}$

$\frac{dy}{dx} = \frac{-x}{2y}$, $\frac{dy}{dx} = \frac{-3x^2}{2y}$ 2

$2y\,dy = -x\,dx$ $\qquad 2y\,dy + 3x^2\,dx = 0$

$y^2 = -\frac{x^2}{2} + c$ $\qquad y^2 + x^3 - c = 0$

$2y^2 + x^2 - c = 0$

$(2y^2 + x^2 - c)(y^2 + x^3 - c) = 0$ $\frac{1}{2}$

2(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$

S.F $(D^2 - 3D + 2)y = \frac{1}{1+e^{-x}}$

A.E $m^2 - 3m + 2 = 0$ $1\frac{1}{2}$

$m = 2, 1$

C.F $= c_1 e^{x} + c_2 e^{2x}$

$y_1 = e^{x}$, $y_2 = e^{2x}$

$u = -\int \frac{y_2 X}{W}$, $V = \int \frac{y_1 X}{W}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix} = e^{3x}$ ①

$u = -\int \frac{e^{-x}}{1+e^{-x}} dx$, $V = \int \frac{1}{e^{2x}(1+e^{-x})} dx$

$= \log(1+e^{-x})$

$V = -e^{-x} + \log|1+e^{-x}|$ ①

$P.I = uy_1 + vy_2$

$= e^{x}\log|1+e^{-x}| - e^{x} + e^{2x}\log|1+e^{-x}|$ ½

$C.S = CF + PI$

$= c_1 e^{x} + c_2 e^{2x} + e^{x}\log|1+e^{-x}| - e^{x} + e^{2x}\log|1+e^{-x}|$

$\frac{dy}{dx} + y\tan x = y^2 \cos x$

dividing by y^2

$y^{-2}\frac{dy}{dx} + y^{-1}\tan x = \cos x$

Put $\frac{1}{y} = z$

$$-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$$ ①

$$-\frac{dz}{dx} + z\tan x = -\cos x$$

$$\frac{dz}{dx} - z\tan x = -\cos x$$

$$P = -\tan x, \quad Q = -\cos x$$

$$I.F = e^{-\int \tan x\, dx} = e^{\log|\cos x|} = \cos x$$ (1½)

Sol^n is

$$z(I.F) = \int Q(IF)\,dx + c$$

$$z\cos x = \int \cos^2 x\, dx + c$$

$$z\cos x = -\frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\boxed{\frac{1}{y}\cos x = -\frac{x}{2} - \frac{\sin 2x}{4} + c}$$ (1½)

3. $$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x\log x + x^2$$

This is Cauchy Homogeneous eqn.

Put $x = e^z$

$z = \log x$

$$\frac{dz}{dx} = \frac{1}{x}$$

Scanned with CamScanner

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x}\frac{dy}{dz}$$

$$x\frac{dy}{dx} = \frac{dy}{dz} = Dy \quad \text{where } D = \frac{d}{dz}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right)$$

$$= -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x^2}\frac{d^2y}{dz^2}$$

$$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz} = (D^2 - D)y$$

$$= D(D-1)y$$

$$\text{IIly} \quad x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y \quad \text{and so on}$$

(1)

eqn (1)

$$D(D-1)y - 3Dy + 5y = z \cdot e^z + (e^z)^2$$

$$(D^2 - D - 3D + 5)y = ze^z + e^{2z}$$

$$(D^2 - 4D + 5)y = ze^z + e^{2z}$$

A.E

$$m^2 - 4m + 5 = 0$$

$$m = 2 \pm i$$

$$C.F = e^{2z}(c_1\cos z + c_2\sin z)$$ $2\frac{1}{2}$

$$P.I = \frac{1}{f(D)} z$$

$$= \frac{1}{D^2-4D+5} \quad z e^{z} + e^{2z}$$

$$= e^{z}\left(\frac{1}{(D+1)^2-4(D+1)+5} z\right) + \frac{1}{D^2-4D+5} e^{2z}$$

$$= e^{z}\left(\frac{1}{D^2-2D+2} z\right) + \frac{e^{2z}}{4-8+5}$$ $3\frac{1}{2}$

$$= e^{z} \frac{1}{2}\left(1+\frac{D^2-2D}{2}\right)^{-1} z + e^{2z}$$

$$= \frac{e^{z}}{2}\left(1-\left(\frac{D^2-2D}{2}\right)\right) z + e^{2z}$$

$$= \frac{e^{z}}{2}(z+1) + e^{2z}$$

$$C.S = CF + PI$$

$$= x^2\left(c_1\cos(\log x) + c_2\sin(\log x)\right) + \frac{x}{2}(\log x+1)$$