QUESTION BANK ON MATRICES

1. Question of 2 marks

Question 1.1. Define Rank of a matrix.

Question 1.2. Find the rank of the following matrices

(a) $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 11 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & -1 & 0 & 5 \\ 0 & 3 & 1 & 4 \end{bmatrix}$

Question 1.3. Define linear dependence and linear independence of vectors and give one example of each.

Question 1.4. State the properties of the orthogonal matrix.

Question 1.5. Prove that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

Question 1.6. State the properties of an eigen values.

Question 1.7. If λ is an eigen value of a non singular matrix A, prove the following:
(i) λ^{-1} is an eigen value of A^{-1}. (ii) $\frac{|A|}{\lambda}$ ia an eigen value of Adj.A (iii) λ^2 is an eigen value of A^2.

Question 1.8. State Cayley Hamilton Theorem.

Question 1.9. Use Cayley Hamilton Theorem, Find the inverse of the

(a) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Question 1.10. Use Cayley Hamilton to find A^5, where $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

2. Question of 4 marks

Question 2.1. Use the rank method to test the consistency of the systems of the equations $4x - y = 12,\ -x - 5y - 2z = 0,\ -2y + 4z = -8$

Question 2.2. For what values of λ and μ do the systems of equations : $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$ have
(i) no solution (ii) unique solution (iii) more than one solution ?

Question 2.3. Find the real value λ for which the system of equations $x+2y+3z=\lambda x,\ 3x+y+2z=\lambda y,\ 2x+2y+z=\lambda z$ have non-trivial solution.

Question 2.4. Discuss the consistency of the following system of equations. Find the solution if consistent

$$\begin{aligned} 5x+3y+7z &= 4 \\ 3x+26y+2z &= 9 \\ 7x+2y+10z &= 5 \end{aligned}$$

Question 2.5. For what values of a and b do the equations $x+2y+3z=6,\ x+3y+5z=9, 2x+5y+az=b$ have (i) no solutions (ii) a unique solution (iii) more than one solution?

Question 2.6. For what values of k the system of equations $x+y+z=2, x+2y+z=-2, x+y+(k-5)z=k$ has no solution ?

Question 2.7. Show that the equations 2x+6y+11=0, 6x+20y-6z+3=0, 6y-18z+1=0 are not consistent.

Question 2.8. Find the eigen values and eigen vectors of the following matrices (a) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

Question 2.9. Using Cayley Hamilton Theorem, Find the inverse of

(a) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

Question 2.10. Reduce the matrix to the normal form and find its rank

(a) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

3. Long Questions

Question 3.1. Show that the matrix $A=\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence find P such that $P^{-1}AP$ is a diagonal matrix,then obtain the matrix $B=A^2+5A+3I$.

Question 3.2. Find a matrix P which transforms the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ into a diagonal form.

Question 3.3. Diagonalise $A=\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and hence find A^8. Find the modal matrix.

Question 3.4. Verify Cayley Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and hence find $B = A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$; Also find $A^{-1} and A^4$