# Quantum mechanics:-    (Bharatdeep Singh, 26, CSE-A, )

$\gamma = e^- + e^+$

$e^+ + e^- = \gamma + \gamma$

## De-Broglie's Hypothesis (matter waves)

Universe → matter; radiation

radiation → particle; wave

particle → Black body Radiation, Photoelectric effect, compton effect.

wave → Interference, Polarization, Difffaraction.

matter → Particle nature. (transfer of momentum)

matter → waves nature (can cross each other),

→ debroglie wave or matter ~~or~~ wave or Pilot wave or ~~by~~ guide wave.

$$\boxed{\lambda = \frac{h}{mv}}$$ ★★★

$$\boxed{E = mc^2}$$ ★★★

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$ ★★★

$$\boxed{K.E = (m - m_0)c^2}$$ ★★

$$\boxed{U = m_0 c^2}$$ ★★

S     S' → $v$

$t$     $t'$

$$\therefore \boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}}$$ ★★★

$$\psi_p = A \sin(\omega t - kx)$$

$\omega$ → angular freq.; $A$ → Amplitude; $k$ → propagation const.

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi\nu$$

## Davison and Germer. Experiment:-

Scanned with CamScanner

$\boxed{\lambda = \frac{h}{mv}}$ $\boxed{E = mc^2}$

$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$ $\boxed{K.E = (m - m_0)c^2}$

$\boxed{U = m_0 c^2}$

S  S' $\longrightarrow v$

$t$  $t'$

$$\boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

angular freq.

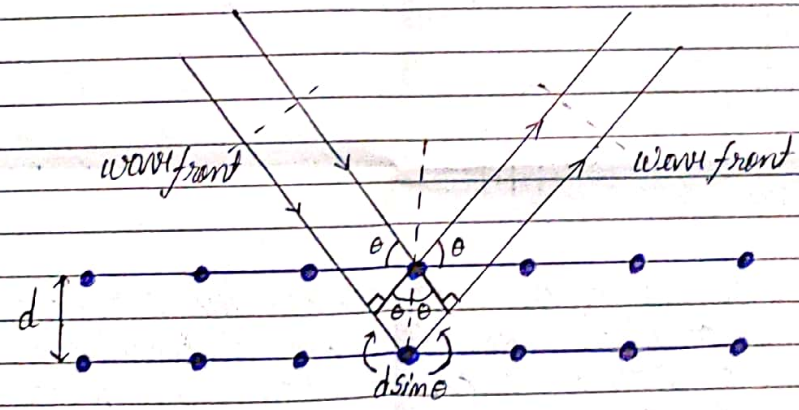
$$\Psi_p = A \sin(\omega t - kx) \qquad k = \frac{2\pi}{\lambda}$$

Amplitude  propagation const.

$= 2\pi\nu$

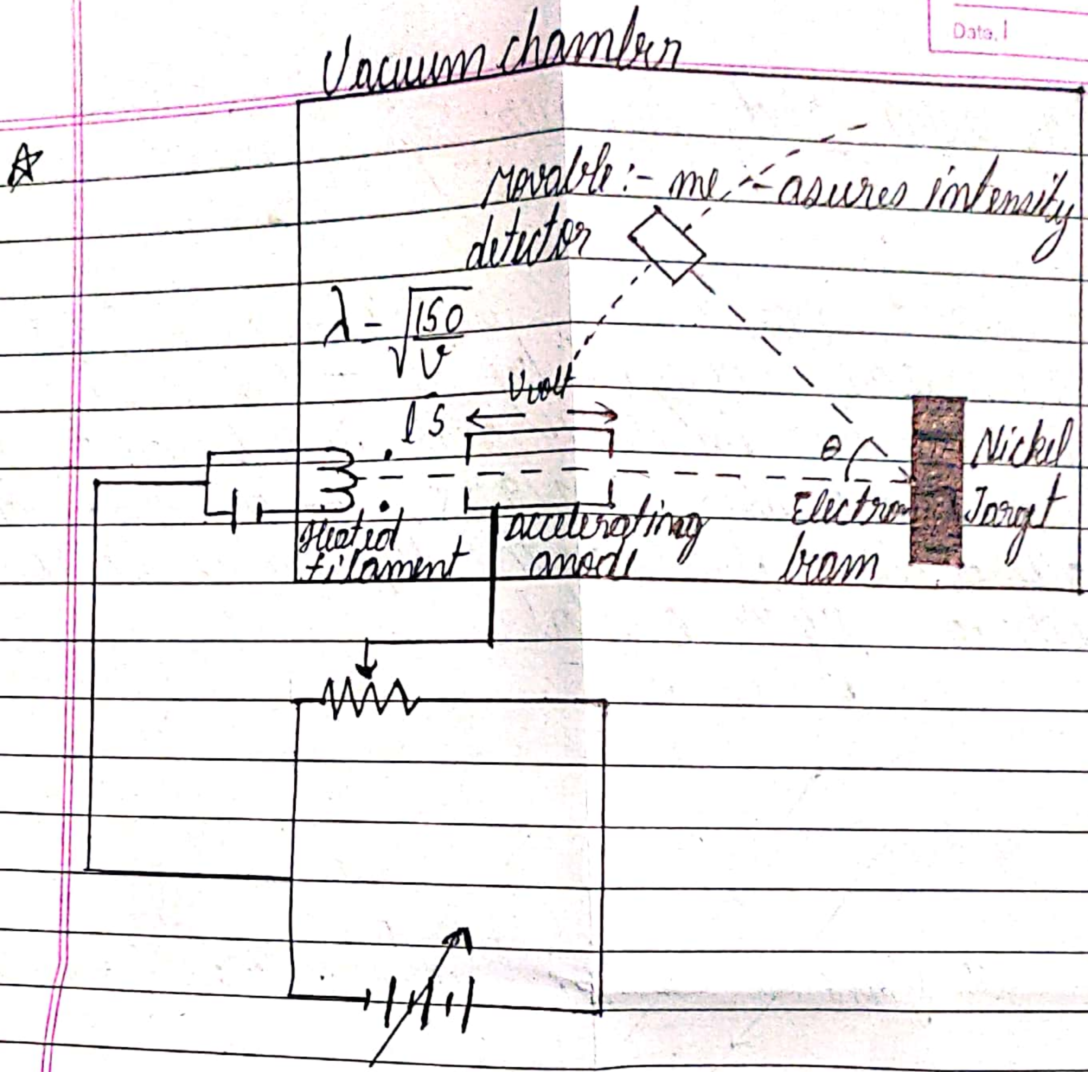
# Davisson and Germer Experiment :-

☆ The wave nature of electrons was first experimentally verified by C.J. Davisson and L.H. Germer in 1927 and independently by G.P. Thomson, in 1928, who observed diffraction effects with beams of electrons scattered by crystal.




$$\Delta x = 2d \sin\theta = n\lambda \quad \text{(Constructive interference)}$$

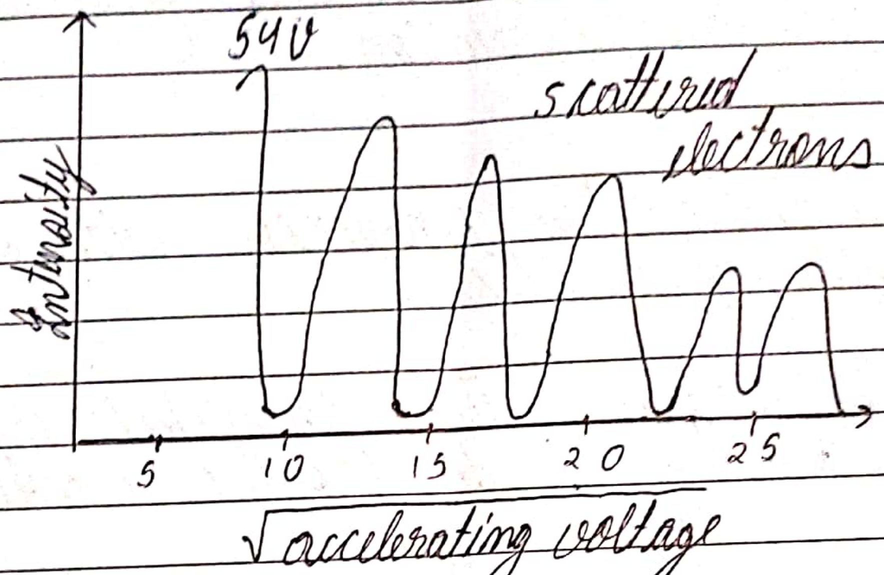
$\Longrightarrow$ high intensity $e^-$ beam




By moving the detector on the circular scale at diff. positions, the intensity of the scattered electron beam is measured for diff. values of angle b/w the incident and scattered e-beams.

The variation of Intensity (I) of the scattered electrons with the angle of scattering is obtained from diff. accelerating voltages.

## ☆ graph:-




☆ The experiment was performed by varying the the accelerating voltage from 44V to 68 V. It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerated volatage of 54 V at a scattering angle $\theta = 50^\circ$.

## # Result:-

From the electron diffraction measurements the wave-length of matter waves was found to be 0.165nm

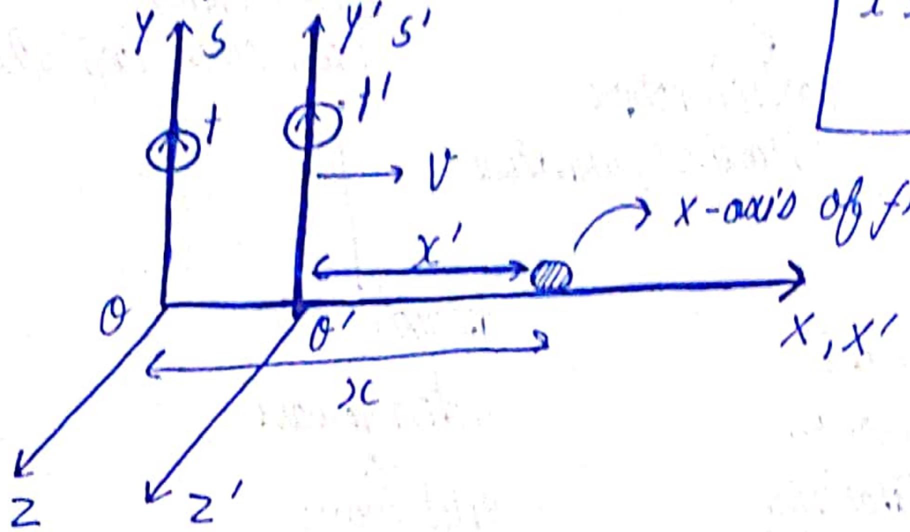
The de-Broglie wavelength $\lambda$ associated with electrons for V = 54V is given by

$$\lambda = \frac{h}{p} = \frac{1.227}{\sqrt{V}} nm$$

$$\boxed{\lambda = \frac{1.227}{\sqrt{54}} = 0.167\,nm}$$

$\Psi_s = A \sin(\omega t) \cos(kx)$

<u>Prove</u>:- Suppose there are two frame of reference S and S' in such a way that their x-axis coincide with each other.




$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}}, \quad \boxed{y' = y, \; z' = z}$$

$$\boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

These 4 eqn. are collectively called Lorentz Transformation.

$m_0 \leftarrow S' \longrightarrow$ rest (object)
$m \leftarrow S \longrightarrow$ motion (object) $\rightarrow v$

$\rightarrow \therefore$ diff. mass in diff. frame of reference.

$\downarrow$
mass of objects acc. to frame of reference.

$x' \longrightarrow$ const.
$x \longrightarrow$ variable.

$$\boxed{m = \frac{m_0}{\sqrt{1 - v^2/c^2}}} \quad \text{--- (1)}$$

we have assumed that the object possesses the wave nature
$\therefore$ wave is observable in frame 'S' and 'S''

$\downarrow$ same wave seen from 'S' must appear to be progressive wave

$\downarrow$ stationary wave.

$$\boxed{\Psi'_s = A' \sin(\omega' t') \cos(k' x')} \quad \text{--- (2)}$$

$\downarrow$ or

$$\boxed{\Psi'_s = A \sin(\omega' t')} \quad \text{--- (3)}$$

$(\because A = A' \cos(k' x'))$

Scanned with CamScanner

$$\boxed{t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}} \longrightarrow ④ \qquad \psi' = \psi$$

$$\Rightarrow \quad \psi = A\sin\left[\frac{\omega'\left(t - \frac{v}{c^2}x\right)}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

in the frame 'S'

$$\psi = A\sin\left[\omega\left(t - \frac{v}{c^2}x\right)\right] \text{ ——— } ⑤ \qquad \boxed{\because \omega = \frac{\omega'}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\psi = A\sin\left[\omega t - \frac{\omega v}{c^2}x\right]$$

$$\psi_p = A\sin(\omega t - kx) \text{ ——— } ⑥ \qquad \left[\because k = \frac{\omega v}{c^2}\right]$$

$V_p$ = phase velocity or wave velocity. $\longrightarrow \dfrac{\text{coeff. of } 't'}{\text{coeff. of } 'x'}$

$$= \nu\lambda$$

$$= \frac{2\pi\nu\lambda}{2\pi} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \frac{\omega}{k}$$

$$\therefore \boxed{V_p = \frac{\omega}{k}} \text{ ——— } ⑦$$

$$V_p = \frac{\omega c^2}{\omega v} \longrightarrow \boxed{V_p = \frac{c^2}{v}} \text{ ——— } ⑧$$

$$V_p = c\underbrace{\left(\frac{c}{v}\right)}_{\text{always greater than '1'}} \qquad V_p > c \text{ (always)}$$

$\therefore$ This means de Broglie waves travels faster than light in vacuum

 (shape of wave associated with object)
(object cannot can come out of wave region)

$\boxed{E = h\nu}$ (wave packets) or (wave group) $\boxed{E = mc^2}$

Since the wave packet belongs to the particle only. $\therefore$ Energy of the wave packet and Energy of the object which is confined inside the wave packet must be same

$$E = h\nu \longrightarrow E = \frac{h\,\nu_p}{\lambda} \longrightarrow E = \frac{h}{\lambda}\frac{c^2}{v} \quad \text{—— (9)}$$
$[\because (8)]$

$$\therefore m\cancel{c^2} = \frac{h}{\lambda}\frac{\cancel{c^2}}{v}$$

$$\boxed{\lambda = \frac{h}{mv}} \longrightarrow \boxed{\lambda = \frac{h}{p}}$$

$\longrightarrow$ (10)

$m$ → mass in motion.

we obtained this result by using the laws of "theory of relativity" so, $\therefore$ eqn. (10) is valid at low speeds as well as high speeds.
$\therefore$ eqn. (10) is a relativistic expression.

Let us take an electron :-

$m_0 = 9.1 \times 10^{-31}$ kg.
$v = 1.6 \times 10^8$ m/sec.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{1}{4}}}$$

$$m = \frac{2m_0}{\sqrt{3}} = \frac{2m_0}{1.732}$$

$$m = \frac{2}{1.732} \times 9.1 \times 10^{-31} = \frac{18.2}{1.732} \times 10^{-31}\ kg.$$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ Js}{\frac{18.2}{1.732} \times 10^{-31} \times 1.5 \times 10^{8}} \approx 0.4 \times 10^{-11}\ m$$

* <u>Wave funtion</u> :-> ( $\Psi$ ) ( changes its value w.r.t. to space & time )

A wave fn. can be defind as a probability amplitude which is given in quantum mechanics, to define state of a particle and see how the particle behave.

eg:- Light waves $\longrightarrow$ Electric, magnetic fields which repeats its value again and again w.r.t. space and time

$\Psi \longrightarrow$ ~~Elect~~ Electric or magnetic field

eg:- Sound waves $\longrightarrow$ ( Pressure or Density of medium )

eg:- Matter waves $\longrightarrow$ ( cannot give physical significance to wave fn $\psi$ )

$\psi$ $\downarrow$ not a measurable quantity but a calculative quantity

whereas $|\Psi|^2 \longrightarrow$ measurable quantity.

$\downarrow$

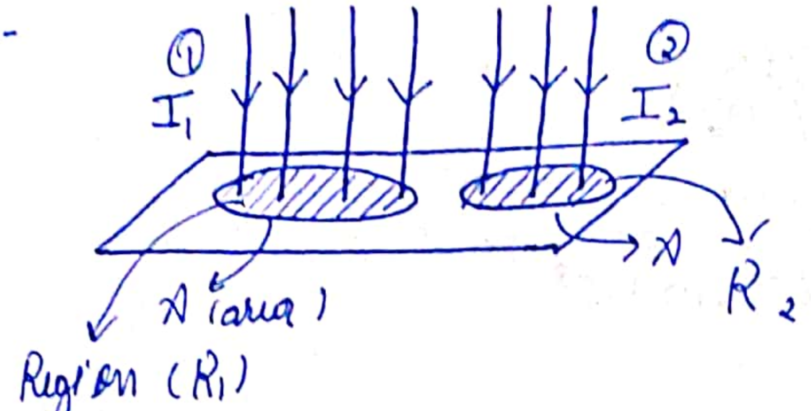
acc. to Max Borne in a small region around any pt. $\therefore |\Psi|^2$ will represent probability of finding the particle whose matter waves are being discussed in a unit region drawn arround. that pt.

 $dV$

$$|\Psi|^2 \propto \frac{dP}{dV}$$

( $dP$ = small Probability of finding a particle in this small region of volume $dV$ )

eg:-




finding photon in $R_1$ and $R_2$

due to $I_1 > I_2$ the probability of finding a photon in $R_1 > R_2$

$\therefore$ finding photon probability $\propto$ Intensity $\propto A^2$ (Amplitude)

$$A^2 = |\Psi|^2 \propto \underset{\text{3-D}}{\frac{dP}{dV}} \text{ or } \underset{\text{2-D}}{\frac{dP}{dA}} \text{ or } \underset{\text{1-D}}{\frac{dP}{dx}}$$

★ <u>Nature of De-broglie waves</u>

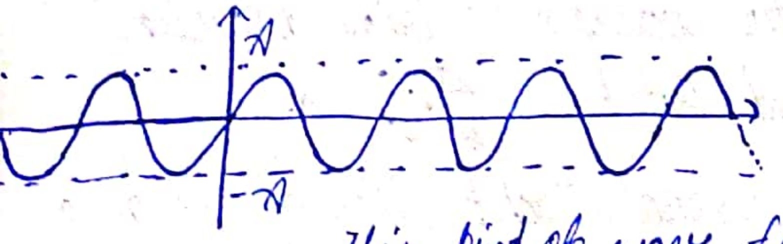
$$\lambda = \frac{h}{mv}$$

$$\Psi = \underline{A}\sin(\underline{\omega}t - \underline{k}x) \quad (\_\ \text{const.})$$

(Plane progressive wave.) +ve x-axis.

range $[-A, A]$
Domain all real numbers

Continuous fn.




$A^2 \rightarrow |\Psi|^2 \propto \frac{dP}{dx}$

1-D

This kind of wave fn cannot be attached to a particle because the particle cannot be present everywhere in the universe at an instant.

2nd reason:- $v_p = \frac{c^2}{v}$ ($v$ = speed of object)

$v_p$ = speed of matter wave associated with object )

$\underbrace{v_p}_{\text{not bounded.}} = c\Big(\frac{c}{\underbrace{v}_{\text{bounded}}}\Big)$ $\therefore$ $\boxed{v_p > c}$ $\boxed{v < c}$ $\rightarrow$ This shows that de-broglie wave will detach from object.

suppose Two plane progressive wave. ?

dual nature is in danger (not possible )

$\Psi_1 = A_1 \sin(\omega_1 t - k_1 x)$ — ①

$+$

$\Psi_2 = A_2 \sin(\omega_2 t - k_2 x)$ — ②

Superimposing ① & ②

$\Psi_1 + \Psi_2 = ?$ ( Somewhere the inteference will be constructive and somewhere destruct-ive )

Then the shape will become :- Probability of finding particle is higher here.


Probability of finding particle is small here.

but still it is infinitely long.

• first the particle was present in the whole universe with equal probability but after superimposing the waves the particle are still present in the whole universe but with unequal Probability

un-desired — desired

★ If we add more ~~than~~ and more progressive waves then the length of the wave will start shrinking

i.e. $\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \ldots\ldots = \Psi$ (wave packet or wave group)

∴ a wave packet or wave group must be attached with an object instead of a de-broglie wave.

∴ we will get


speed smaller than 'C'

delocalized here only.

It has been formed by supperimposing of waves. with ~~speeds~~ $V_1, V_2, V_3 \ldots\ldots V_n$. all greater than 'C'. → velocities.

∴ but always. $\underset{>C}{V_1} + \underset{>C}{V_2} + \underset{>C}{V_3} + \ldots\ldots + \underset{>C}{V_n} = \underset{<C}{V_g}$


~~or~~ diff. direction diff waves

$V_g = \sqrt{c^2 + c^2 + 2c^2\left(-\frac{1}{2}\right)}$ vector addn

$= c$

Scanned with CamScanner

# Formation of wave packet:-

Power & programs.
N N Bhu
Basic electr.
and linear.
circuit.

$$\psi_1 = A \sin(\omega t - kx) \quad \text{--- (1)}$$

$$\psi_2 = A \sin[(\omega + d\omega)t - (k + dk)x] \quad \text{--- (2)}$$

$$\psi = \psi_1 + \psi_2$$

$$= A\{\sin \underbrace{[(\omega + d\omega)t - (k+dk)x]}_{C} + \sin \underbrace{(\omega t - kx)}_{D}\}$$

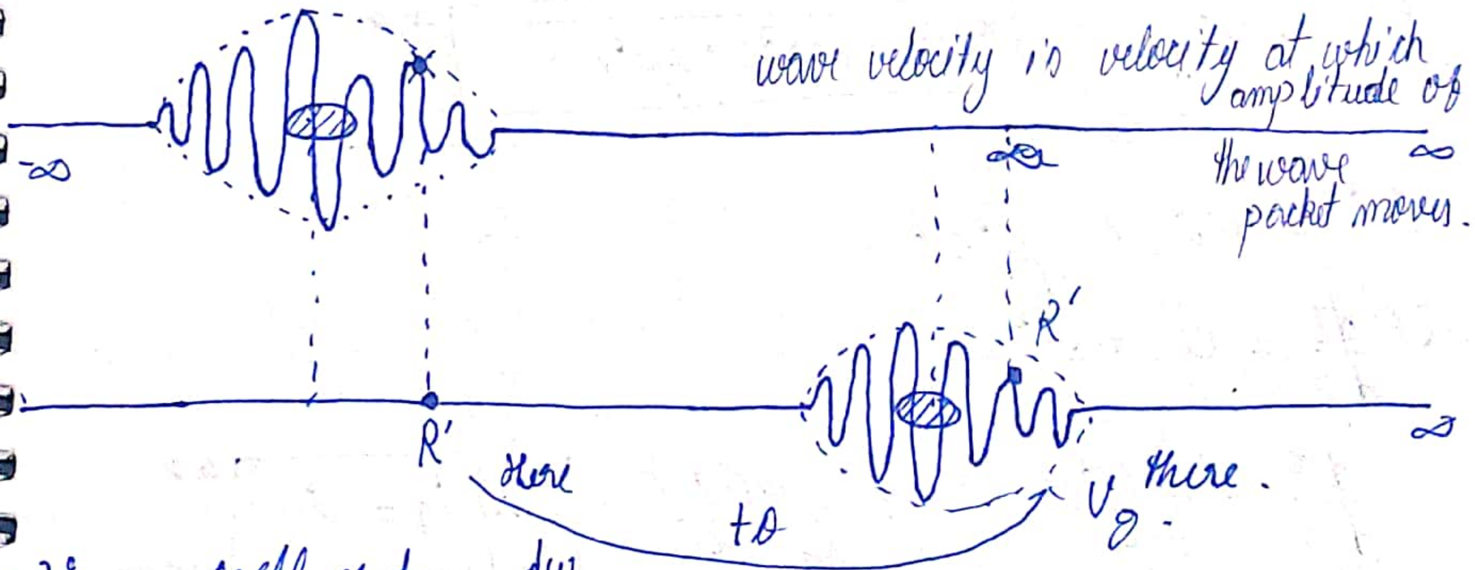
$$= 2A \sin\left[\left(\omega + \frac{d\omega}{2}\right)t - \left(k + \frac{dk}{2}\right)x\right] \cos\left[\frac{d\omega}{2}t - \frac{dk}{2}x\right] \quad \text{--- (3)}$$

Put $2A \cos\left(\frac{d\omega}{2}t - \frac{dk}{2}x\right) = R$ --- (4)

$d\omega$, and $dk$ are small.

$$\psi \approx R \sin(\omega t - kx) \quad \text{--- (5)}$$

(Amplitude is variable
$[R = f(t, x)]$




wave velocity is velocity at which amplitude of the wave packet moves.

$$V_{g_R} = \frac{\text{coeff. of } t}{\text{coeff. of } x} = \frac{\frac{d\omega}{2}}{\frac{dk}{2}} = \frac{d\omega}{dk}$$

$$\therefore \boxed{V_g = \frac{d\omega}{dk}} \quad \text{—— (6)}$$

$$\boxed{V_{av} = \frac{\omega + d\omega - \omega}{k + dk - k} = V_g = \frac{d\omega}{dk}}$$

we know that: $E = h\nu = \frac{h}{2\pi} \times 2\pi\nu = \hbar\omega$

$$\therefore \boxed{E = \hbar\omega} \quad \text{—— (7)} \qquad \hbar = 1.054 \times 10^{-34}\ \text{J sec.}$$

$\downarrow$

Planck's reduced const.

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k$$

$$\Rightarrow \boxed{p = \hbar k} \quad \text{—— (8)}$$

7, 8 are called einstein's de broglie's eq^n.

$$V_g = \frac{d\omega}{dk} = \frac{\hbar}{\hbar}\frac{d\omega}{dk} = \frac{d(\hbar\omega)}{d(\hbar k)}$$

$$\boxed{V_g = \frac{dE}{dp}} \quad \text{—— (9)}$$

$$\boxed{E = mc^2} \quad \text{—— (10)} \qquad , \boxed{E = \sqrt{m_0^2 c^4 + p^2 c^2}} \quad \text{—— (11)}$$

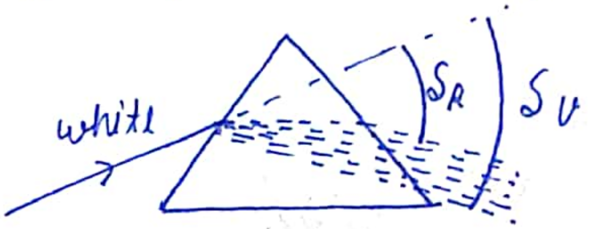
$$E^2 = m_0^2 c^4 + p^2 c^2$$

$$2E \frac{dE}{dp} = 0 + 2c^2 p$$

$$\frac{dE}{dp} = \frac{\cancel{2}c^2 p}{\cancel{2}E} = \frac{c^2(mv)}{mc^2}$$

$$\therefore \boxed{V_g = v} \quad \text{—— (12)}$$

# Dispersion relation for de-broglie waves :-




$$\mu = C_1 + \frac{C_2}{\lambda_2} + \frac{C_3}{\lambda_3} \cdots\cdots \infty$$

speed of light in medium $= \frac{c}{\mu}$

$$\mu \approx C_1 + \frac{C_2}{\lambda_2}$$ , ~~If~~ $\lambda \uparrow \Rightarrow \mu \downarrow \Rightarrow$ Speed$\uparrow$

$$\delta = (\mu - 1) A$$

We know that :- $v_p = \frac{\omega}{k}$ , $\omega = k v_p$

$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(k v_p)$$

$$v_g = v_p + k\frac{dv_p}{dk}$$

$$= v_p + \frac{dv_p}{d\lambda} \times \frac{d\lambda}{dk}$$

$k = \frac{2\pi}{\lambda}$

$\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$

$$v_g = v_p - \frac{\cancel{2\pi}}{\lambda}\frac{dv_p}{d\lambda}\left(\frac{\lambda^2}{\cancel{2\pi}}\right)$$

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}} \longrightarrow (13)$$

dispersion relation.

Case:- I $v_p$ is independent of 'λ'

i.e. No dispersion at all

$\therefore \frac{d v_p}{d\lambda} = 0$ $\therefore v_g = v_p$ ! ✗ reject.

$v < c$ , $\frac{c^2}{v} > c$

Case:- II $\frac{d v_p}{d\lambda} = -ve$ (dec. fn of λ)

$\lambda\uparrow \Rightarrow v_p \downarrow$ ($\neq$ light) i.e. anomalous dispersion.

$v_g = v_p + \underbrace{(\text{Something})}_{+ve} \lambda$

$v_g > v_p$ , however $v_p > c$

$\therefore v_g > c$ ! ✗ reject.

but $\boxed{v_g = v}$ ✓

Case:- III $\frac{d v_p}{d\lambda} = +ve$ (↑ fn. of λ)

$\lambda\uparrow \Rightarrow v_p \uparrow$ (= light) i.e. normal dispersion.

$\therefore v_g = v_p - \underbrace{(\text{Something})}_{+ve} \lambda$

$\Rightarrow v_g < v_p$ we need $v_g < c$

Scanned with CamScanner

we want

$v_g < c$

$v_p - \lambda \frac{dv_p}{d\lambda} < c$

or

$v_p - c < \lambda \frac{dv_p}{d\lambda}$

or $\boxed{\frac{dv_p}{d\lambda} > \frac{v_p - c}{\lambda}}$ ans

# de-Broglie wavelength of a charged particle:-

## Relativistic case:-

$q$ = charge , $v$ = speed gained. , $V$ = accelerating potential.

$m$ = mass in motion. , $p$ = linear momentum , $m_0$ = rest mass

$\lambda = \frac{h}{mv} = \frac{h}{p}$ —— ①

initial vel = $u = 0$

$W$ = gain in K.E (T)

$= E - u$

work done = $qV$ —— ②

Change in K.E. = final K.E. - initial K.E.

= Total energy - rest mass energy.

$= E - m_0c^2$

$= \sqrt{m_0^2c^4 + p^2c^2} - m_0c^2$

—— ③

work done = change in K.E.

$$qV = \sqrt{m_0^2c^4 + p^2c^2} - m_0c^2$$

$$qV + m_0c^2 = \sqrt{m_0^2c^4 + p^2c^2}$$

$$(qV + m_0c^2)^2 = m_0^2c^4 + p^2c^2$$

$$q^2V^2 + \cancel{m_0^2c^4} + 2qVm_0c^2 = \cancel{m_0^2c^4} + p^2c^2$$

$$p^2c^2 = q^2V^2 + 2qVm_0c^2$$

$$p^2 = \frac{q^2V^2}{c^2} + \frac{2qVm_0\cancel{c^2}}{\cancel{c^2}}$$

$$p^2 = \frac{q^2V^2}{c^2}\left(1 + \frac{2m_0c^2}{qV}\right)$$

$$p = \frac{qV}{c}\sqrt{1 + \frac{2m_0c^2}{qV}}$$

$$\lambda = \frac{h}{p}$$

$$\therefore \boxed{\lambda = \frac{hc}{qV\sqrt{1 + \frac{2m_0c^2}{qV}}}}$$

→ applicable for any particle at any speed

★ Non-relativistic case (accelerating potential is low, i.e. speed gained by the particle is less than the speed of light)

$$K.E. = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

initial velocity $= u = 0$

$\therefore$ change in $K.E. = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{p^2}{2m}$$

work done $= qV$

$$qV = \frac{p^2}{2m}$$

$$p^2 = 2mqV$$

$$p = \sqrt{2mqV}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mqV}}}$$

Particle:→

(i) It can't be omnipresent. So a single debroglie wave cannot be attached to a particle.

(ii) Debroglie wave travels faster than c, object less than 'c' $\therefore$ wave detach.

(iii) Probability of finding particle will be equal in the whole universe.

$\therefore$ A single debroglie wave cannot be attached to a particle.

if charged particle is $e^{\theta}$/positron.

$q = 1.6 \times 10^{-19}\ C$

$m = 9.1 \times 10^{-31}\ kg.$

$h = 6.6 \times 10^{-34}\ Js$

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}}\ m$$

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

or $\lambda = \sqrt{\frac{150}{V}}\ \text{Å}$

(at low speed)

Scanned with CamScanner

if External energy $\approx$ rest mass energy.

Then speed $(v) \approx c$

10% of rest mass energy

$$= \frac{10}{100} \times m_0 c^2$$

$$= \frac{m_0 c^2}{10}$$

for external energy given $\leq 0.1\, m_0 c^2$ => (non-relativistic)

" " " " $> 0.1\, m_0 c^2$ => (relativistic)

for electron $m_0 c^2 = 511\ keV$

$$\frac{0.1\, m_0 c^2}{e} = \frac{51.1\ keV}{e}$$

$= 51.1\ kV \geq$ accelerating potential

then $\lambda = \frac{12.27}{\sqrt{V}}\ \mathring{A}$

for Proton, $m_0 c^2 = 931.5\ MeV$

$$\frac{0.1\, m_0 c^2}{e} = \frac{93.15\ MeV}{e}$$

$= 93.15\ MV \geq$ accelerating potential.

then $\lambda = \frac{h}{\sqrt{2mqV}}$

if $e^{\circ}$ beam is accelerated by 100 V

$$\therefore \lambda = \frac{12.27}{\sqrt{100}} \mathring{A}$$

$$\lambda = 1.227 \mathring{A}$$

$$\sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10^{6}}{9.1 \times 10^{-31}}}$$

$$4\sqrt{2 \times 10^{-20+6+31}}$$

$$\frac{4}{3}\sqrt{2 \times 10^{18}}$$

if $e^{\circ}$ beam is accelerated by $10^{6}$ V

$$\boxed{V = pc\sqrt{\frac{1}{m_0^2 c^2 + p^2}}}$$

$$\lambda = \frac{hc}{qV\sqrt{1 + \frac{2m_0 c^2}{qV}}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 10^{6}\sqrt{1 + \frac{2 \times 9.1 \times 10^{-31}(3 \times 10^{8})^2}{1.6 \times 10^{-19} \times 10^{6}}}}$$

# ~~Dt~~ De-broglie wavelength of a thermal particle:-

$$K.E. = \text{Thermal } K.E. = \frac{3}{2}kT$$

$$\boxed{k = 1.23 \times 10^{-38}\ J/K}$$

$$\frac{1}{2}mv^2 = \frac{3}{2}kT$$

$$m = 1.66 \times 10^{-27}$$

$$m_p = 1840\, m_e$$

$$\frac{p^2}{2m} = \frac{3}{2}kT$$

$$p^2 = 3mkT$$

$$p = \sqrt{3mkT}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mkT}}}$$

T = Kelvin

Scanned with CamScanner

☆ Electron $V = 11.24 \times 10^5 V$

$$\lambda = \frac{12.27\ \text{Å}}{\sqrt{11.24 \times 10^5}} = 1.15 \times 10^{-2}\ \text{Å} = 1.15\ pm.$$ (wrong answer)

Energy given $= qV$

$= e \times 11.24 \times 10^5\ V$

$= 1124 \times 10^3\ V$

$= 1124\ kV \quad >> 511\ kV$

$$K.E. = (m - m_0)c^2$$

$$= \left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0\right)c^2$$

$$qV = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\cancel{e} \times 11.24 \times 10^5 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \times (511\ k\cancel{e}V)$$

$$v = 2.85 \times 10^8\ m/sec. \quad < c$$

$$\lambda = \frac{hc}{qV\sqrt{1 + \frac{2m_0c^2}{qV}}}$$

$$\boxed{\lambda = 0.80\ pm}$$

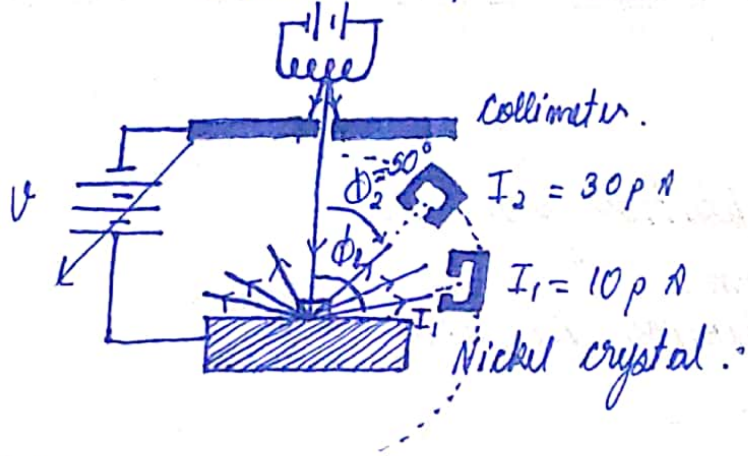
$$2\pi r = n\lambda$$

$$2\pi r = n\frac{h}{mv}$$

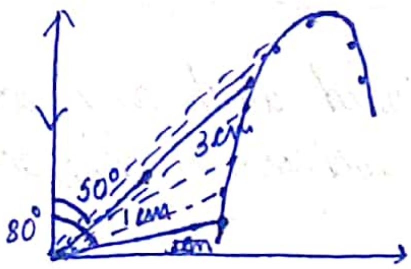
$$mvr = \frac{nh}{2\pi}$$

$$\boxed{L = \frac{nh}{2\pi}}$$

# Davison and Germer experiment:-




Incident beam:-

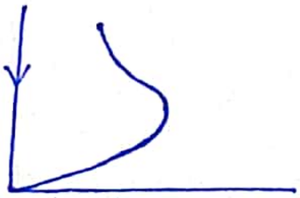



scale 1cm = 10pA

Bragg's law: The diffraction from a crystal of n th order follows:-

$$\boxed{2d\sin\theta = n\lambda.}$$


$V = 54,\ \phi = 50°$



$V = 44$

$V = 48V.$

Scanned with CamScanner

$V = 64$

$n = 1$
$d = 0.91\,Å$
$\theta = 65°$

$2 \times 0.91 \sin 65° = 1 \times \lambda$

$$\boxed{\lambda = 1.65\,Å}$$

$$\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67\,Å$$

# Comparison b/w de-Broglie waves and Electromagnetic waves:

I. Similarities:->

1. Both waves show superposition principle.
2. Can pass through vaccum.
3. Both show normal dispersion (i.e. $\lambda\uparrow \Rightarrow v_p\uparrow$)

II. Differences:->

| Electromagnetic waves :-> | De-broglie waves:-> |
|---|---|
| 1. associated only with accelerating charged particle | 1. associated with any particle at rest, uniform motion, accelerating. |
| 2. Not deflected by electric & magnetic field | 2.  the wave packet is always associated with the particle and the particle is always associated with wave group. ∴ The de-broglie waves may or may not be deflected by the Electric and magnetic field depending on the nature of the particle |

| | |
|---|---|
| 3. Travels in vacuum with speed 'c' $3 \times 10^8$ m/sec | 3. $v_p = \frac{c^2}{v} > c$ <br> $v_g = v < c$ |
| 4. Do not show dispersion in vacuum or show dispersion in medium only of all EM waves of diff. $\lambda/\nu$ travels with same speed in vacuum | 4. They show dispersion in vacuum as well as in medium. |

# Basis for schrodinger

$\Psi \longrightarrow$ calculatable

$\longrightarrow$ not measurable / observable

$\longrightarrow$ has no significance in itself.

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\Psi = 0} \longrightarrow \text{1-D schrodinger eqn.}$$

or

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-U)\Psi = 0}$$

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0} \qquad \boxed{\Psi = y \ , \ x = t \ , \ \frac{8\pi^2 m}{h^2}(E-U) = \omega^2}$$

soln. $\boxed{y = A\sin\omega t + B\cos\omega t}$ $\qquad \boxed{\omega = \sqrt{\frac{k}{m}}}$

Scanned with CamScanner

$\therefore \Psi = f(x, t)$

for 3-D $\longrightarrow$ $\Psi = f(x, y, z, t)$ $\therefore$ wave fn. is calculatable

In 1-D $\longrightarrow$ $\Psi = \dfrac{1}{\sqrt{\text{length}}}$ $\longrightarrow$ so it cannot be measured

Although $\Psi$ has no significance and it is not even measurable then why it is useful?

# Operators in quantum mechanics:->

$\downarrow$

Any process which changes one fn. to another fn.

let $\hat{A}$ is an operator.

$\hat{A} = \sqrt{\phantom{x}}$ = square root of ----- $\rightarrow$ no significance.

$\hat{A}(2) = \sqrt{2} = 1.414$

$\hat{A}(3) = \sqrt{3} = 1.732$

$\hat{A}(4) = \sqrt{4} = 2$

$\hat{A}^2(4) = \sqrt{\sqrt{4}}$

$= \sqrt{2} = 1.414$

$\hat{A}(\Psi) = \sqrt{\Psi}$

$\hat{A}(4x^2) = \sqrt{4x^2} = 2x$

let $\hat{A} = \dfrac{d}{dx}$ = diff. of ----- w.r.t. '$x$'

$\Psi = \sin 4x$

$\therefore \hat{A}(\psi) = \frac{d(\psi)}{dx}$ = diff. of $\sin 4x$ w.r.t. $x$

$= \frac{d(\sin x)}{dx} = 4\cos 4x$

let $\hat{B} = \frac{d^2}{dx^2} = \left(\frac{d}{dx}\right)\left(\frac{d}{dx}\right)$

$= (\hat{A})(\hat{A}) = (\hat{A})^2$

$\therefore \hat{B}(\psi) = \hat{A}^2(\psi) = \frac{d^2}{dx^2}(\psi) = \frac{d^2}{dx}(\sin 4x)$

$= -16 \sin 4x$

- Linear operator :-

$$\boxed{\hat{A}(C_1\psi_1 + C_2\psi_2) = C_1\hat{A}\psi_1 + C_2\hat{A}\psi_2}$$

let, $\hat{A} = \frac{d}{dx}$

all diff. operator.
" integral " .

$\frac{d}{dx}(\psi_1 \pm \psi_2) = \frac{d\psi_1}{dx} \pm \frac{d\psi_2}{dx}$

$\hat{A} = \sqrt{\phantom{x}}$

$\hat{A}(4) = \sqrt{4} = 2$

$\hat{A}(9) = \sqrt{9} = 3$

$\hat{A}(4+9) = \sqrt{13} = 3.5$

i.e. $\hat{A}(4+9) \neq \hat{A}(4) + \hat{A}(9)$
$\therefore$ (non linear fn)

The operators that we use in quantum mechanics are linear operators:->

☆ Eigen value eqn.:->
- Eigen fn.
- Eigen operator.
- Eigen value

Definition:-

$$\boxed{\hat{A}(\Psi) = \alpha \Psi}$$

$\Psi$ → Eigen fn

$\hat{A}(\Psi) = \alpha\Psi$ → Eigen value eqn.

$\hat{A}$ → Eigen operator.

$\alpha$ → const. = real or imaginary.

(Secular eqn.)

example:-> let $\hat{A} = \frac{d^2}{dx^2}$, $\Psi = \sin 4x$

$\hat{A}(\Psi) = -16 \sin 4x$

$$\boxed{\hat{A}(\Psi) = -16\ \Psi}$$

$\hat{A} = \frac{d^2}{dx^2}$, $-16$ = const., $\Psi$ = Eigen fn.

$\hat{A} = \frac{d}{dx}$, $\Psi = e^{4x}$

$$\boxed{\begin{aligned}\hat{A}(\Psi) &= \frac{d}{dx}(e^{4x}) = 4e^{4x} \\ &= 4\Psi\end{aligned}}$$

$\Psi = \sin 4x$, $\hat{A} = \frac{d}{dx}$

$\hat{A}(\Psi) = \frac{d}{dx}(\sin 4x) = 4 \cos 4x$

$$\boxed{\hat{A}(\Psi) = 4\Psi'}$$

The quantum mechanical operators are always fixed so, we have to choose an appropriate of wave fn so, that the Eigen value eqn. holds.

$\boxed{\hat{A}(\Psi) = \tilde{\alpha}\Psi}$ matrix eqn.

Scanned with CamScanner

Let $\Psi = A e^{-i(\omega t - \vec{k}\cdot\vec{r})}$ ——— ①

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$

$\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z$

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{p}_x(\Psi) = -i\hbar\frac{\partial \Psi}{\partial x}$$

$$= -i\hbar\frac{\partial}{\partial x}\{A e^{-i(\omega t - \vec{k}\cdot\vec{r})}\}$$

$$= \left[-i\hbar A e^{-i(\omega t - \vec{k}\cdot\vec{r})}\right] \times \frac{\partial}{\partial x}\left[-i(\omega t - \vec{k}\cdot\vec{r})\right]$$

$$= [-i\hbar\Psi](-i)[0 - (k_x(1) + 0 + 0))]$$

$$= +k_x\hbar\Psi$$

$[\because p = \hbar k$
$p_x = \hbar k_x$

$$\boxed{\hat{p}_x(\Psi) = p_x \Psi} \rightarrow \text{Eigen value eqn.}$$

operator. Const. Eigen fn.

$$\hat{p}_y = -i\hbar\frac{\partial}{\partial y}$$

$$\therefore \boxed{\hat{p}_y(\Psi) = p_y \Psi}$$

similarly.

$$\boxed{\hat{p}_z(\Psi) = p_z \Psi}$$

$$\boxed{\hat{p} = -i\hbar \vec{\nabla}}$$

$$\hat{p} = \hat{i}\left(-i\hbar\frac{\partial}{\partial x}\right) + \hat{j}\left(-i\hbar\frac{\partial}{\partial y}\right) + \hat{k}\left(-i\hbar\frac{\partial}{\partial z}\right)$$

$$\boxed{\hat{p} = \hat{i}\hat{p}_x + \hat{j}\hat{p}_y + \hat{k}\hat{p}_z}$$

$-i\,i\,\hbar k_x \Psi$

$p_x \Psi$

$$\hat{p}(\Psi) = \hat{i}\,\hat{p}_x(\Psi) + \hat{j}\,\hat{p}_y(\Psi) + \hat{k}\,\hat{p}_z(\Psi)$$

$$\hat{p}(\Psi) = (\hat{i}p_x + \hat{j}p_y + \hat{k}p_z)\Psi$$

$$\boxed{\hat{p}(\Psi) = \vec{p}\,\Psi}$$

☆ <u>Hamiltonian operator</u>:-> $(\hat{H})$ ⟶ (time independent form.)

time dependent form.

$$\hat{H} = i\hbar\frac{\partial}{\partial t}$$

$$\hat{H}(\Psi) = i\hbar\frac{\partial}{\partial t} A e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

$$\hat{H}(\Psi) = i\hbar\left[A e^{-i(\omega t - \vec{k}\cdot\vec{r})}\right](-i\omega)$$

$$\boxed{\hat{H}(\Psi) = \hbar\omega\Psi} \quad \therefore \boxed{\hat{H}\Psi = E\Psi}$$

$$\boxed{\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + U}$$

$$\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

<u>time independent</u>:-

$$\hat{H}(\Psi) = \frac{-\hbar^2}{2m}\nabla^2(\Psi) + U(\Psi)$$

$$\nabla^2(\Psi) = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$

Scanned with CamScanner

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x}\{A e^{-i(\omega t - \vec{k}.\vec{r})}\}$$

$$= \{A e^{-i(\omega t - \vec{k}.\vec{r})}\} \frac{\partial}{\partial x}(-i(\omega t - \vec{k}.\vec{r}))$$

$$= -i\Psi\{(0 - (k_x(1) + 0 + 0))\}$$

$$= i k_x \Psi$$

$$\frac{\partial \Psi}{\partial x} = i k_x A e^{-i(\omega t - \vec{k}.\vec{r})}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = i k_x \frac{\partial \Psi}{\partial x}$$

$$= i(k_x)(i k_x \Psi)$$

$$\boxed{\frac{\partial^2 \Psi}{\partial x^2} = -k_x^2 \Psi}$$

$$\therefore \boxed{\frac{\partial^2 \Psi}{\partial y^2} = -k_y^2 \Psi}$$

$$\therefore \frac{\partial^2 \Psi}{\partial z^2} = -k_z^2 \Psi$$

$$\nabla^2(\Psi) = -(k_x^2 + k_y^2 + k_z^2)\Psi$$

$$= -k^2 \Psi$$

$$\therefore \hat{H}(\Psi) = \frac{-\hbar^2}{2m}(-k^2(\Psi)) + U(\Psi)$$

$$= \frac{\hbar^2 k^2}{2m}\Psi + U(\Psi)$$

$$\boxed{p = \hbar k}$$

$$= \frac{p^2}{2m}\Psi + U\Psi$$

Scanned with CamScanner

$\hat{H}(\Psi) = \left(\frac{p^2}{2m} + U\right)\Psi$

$\boxed{K.E + P.E = E}$

$\therefore \boxed{\hat{H}(\Psi) = E\Psi}$ → The wave fn. is also called state fn. because wave fn. gives all physical parameter of the physical state.

# Position operator:→

$$\boxed{\begin{aligned} \hat{x} &= x \\ \hat{y} &= y \\ \hat{z} &= z \end{aligned}}$$

$$\boxed{\hat{R} = (x\hat{i} + y\hat{j} + z\hat{k})}$$

# Potential energy operation:-

$$\boxed{\hat{U} = U}$$

(never depends upon velocity, momentum) or any derivative $\frac{d}{dt}$

for 1-D Harmonic oscillator.

$$\boxed{\hat{U} = \frac{1}{2}kx^2}$$

$E = \frac{p^2}{2m} + U$

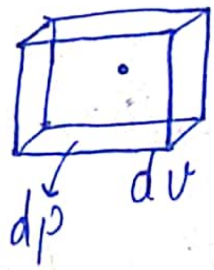
$\frac{dE}{dp} = \frac{p}{m} + 0$

# Kinetic energy operator:-

$$\hat{T} = \frac{-\hbar^2}{2m}\nabla^2$$

# Normalization of a wave fn.:-

$$\boxed{|\Psi|^2 \propto \frac{dP}{dV}}$$




$\frac{dP}{dV}$ = Probability density.

$$\Rightarrow \boxed{|\Psi|^2 = N\frac{dP}{dV}} \quad \text{—— ①}$$

If $N = 1$

$\Rightarrow |\Psi|^2 = \frac{dP}{dV}$

i.e. $|\Psi|^2 \rightarrow$ exactly equal to probability density.

Then $\rightarrow$

$\Psi \rightarrow$ Normalized W.F

$$\frac{dP}{dV} \propto |\Psi|^2$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dP = N|\Psi|^2 dV = 1$$

if $|\Psi|^2 \neq \frac{dP}{dV}$ or $N \neq 1$

Then $\rightarrow$

$\Psi \rightarrow$ Unnormalized W.F.

It is always possible to obtain normalized W.F. from unnormalized W.F by multiplying $\Psi$ with some suitable const..

W.F. = $\Psi \rightarrow$ Soln of Schrödinger eqn. (S.E.)

S.E. $\rightarrow$ differential eqn.

That if $\Psi$ is a soln. of a differential eqn. then $c\Psi$ is also a soln. of diff. eqn.

Let $\Psi$ be the soln. ~~of~~ obtained by solving a S.E.

Let $|\Psi|^2 \rightarrow$ known

Find $\int_{-\infty}^{\infty} |\Psi|^2 dV$

Let $\int_{-\infty}^{\infty} |\Psi|^2 dV = N$

$$\Rightarrow \int_{-\infty}^{\infty} \left|\frac{\Psi}{\sqrt{N}}\right|^2 dV = 1$$

Put $\frac{\Psi}{\sqrt{N}} = \Psi'$

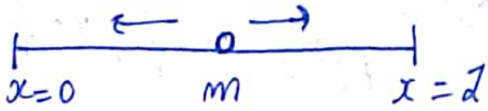
$$\therefore \int_{-\infty}^{\infty} |\Psi'|^2 dV = 1$$

$$\therefore \boxed{|\Psi'|^2 = \frac{dP}{dV}}$$

Ans

Scanned with CamScanner

$\Psi' = \frac{\Psi}{\sqrt{N}}$ is normalised




$\Psi = A \sin\left(\frac{n\pi x}{L}\right)$ (given)

we assert that 'A' is chosen in such a way that

$|\Psi|^2 = \frac{dp}{dx}$ (exactly)

$\Rightarrow dP = |\Psi|^2 dx$

$\int_0^L dP = 1$

$\rightarrow \int_0^L |\Psi|^2 dx$

$\rightarrow \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$

$\frac{A^2}{2} \int_0^L \left[1 - \cos\left(\frac{2n\pi x}{L}\right)\right] dx$

$\frac{A^2}{2} [L - 0] = 1$

$\therefore A = \sqrt{\frac{2}{L}}$

$\therefore \boxed{\Psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$

Scanned with CamScanner

# Hermition Operation:->

$$\boxed{\int_{-\infty}^{\infty} \psi_m^* \hat{A} \psi_n \, dV = \int_{-\infty}^{\infty} \psi_m (\hat{A} \psi_n)^* \, dV} \quad \text{①}$$

**Importance:->**

Let $\hat{A}$ is Hermition operator.

$\therefore$ $\hat{A}$ satisfies eqn — ①

Now $\psi_m = \psi_n$ (say)

$$\int_{-\infty}^{\infty} \psi_m^* \hat{A} \psi_n \, dV = \int_{-\infty}^{\infty} \psi_m (\hat{A} \psi_n)^* \, dV$$

$$\Rightarrow \int_{-\infty}^{\infty} \psi_n^* \hat{A} \psi_n \, dV = \int_{-\infty}^{\infty} \psi_n (\hat{A} \psi_n)^* \, dV$$

Let $\hat{A}(\psi_n) = \alpha_n \psi_n$

$$\Rightarrow \int_{-\infty}^{\infty} \psi_n^* \alpha_n \psi_n \, dV = \int_{-\infty}^{\infty} \psi \, \psi_n^* \alpha_n^* \, dV$$

$$\alpha_n \int_{-\infty}^{\infty} \psi_n^* \psi_n \, dV = \alpha_n^* \int_{-\infty}^{\infty} \psi_n \psi_n^* \, dV$$

$$\alpha_n \int_{-\infty}^{\infty} |\psi_n|^2 \, dV = \alpha_n^* \int_{-\infty}^{\infty} |\psi_n|^2 \, dV$$

$\therefore (\alpha_m - \alpha_m^*) = 0$ or $\int_{-\infty}^{\infty} |\Psi_m|^2 dV = 0$ (rejected)

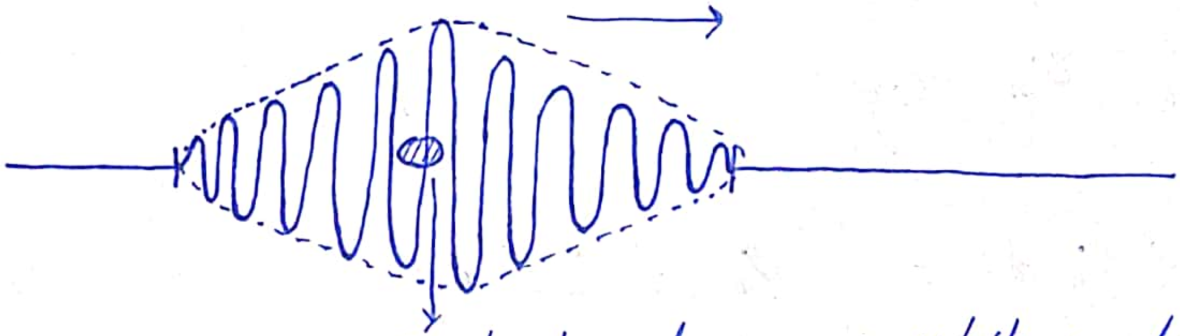
$\frac{\partial}{\partial t}$ = explicit fn

$\frac{d}{dt}$ = implicit fn

$\boxed{\alpha_m = \alpha_m^*}$ $\therefore \alpha_m$ = a real no.

Eigen values of Hermition operators are always real.

# Wave mechanical Treatment :->



can the travel in any arbitrary direction. inside the wave packet.

The wave mechanical treatment is that method in which we study the average motion of a particle by observing the motion of wave packet attached with it

| Newtonian | wave mechanical: |
| --- | --- |
| x, V or p directly measured with 100% accuracy (deterministic approach) | we study the motion of wave packet (gives us the average motion of the particle) (undeterministic approach) or (Probalistic " ) |

# Schrödinger Equation :- (eqn. of motion of a wave packet attached with a particle)

Time independent SE (TISE)

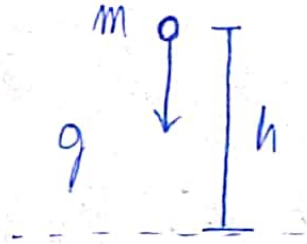
$$\boxed{\Psi(\vec{x},\vec{p},t) = \psi(\vec{x},\vec{p})\,\phi(t)}$$

is possible when Potential energy 'U' does not depend upon 't' explicitely.

Time dependent SE (TDSE)

$$\boxed{\Psi(\vec{x},\vec{p},t) \neq \psi(\vec{x},\vec{p})\,\phi(t)}$$

example (Josephson effect)
Potential energy depends explicitly on 't'




$U = mgh$

'U' is $\downarrow$ with $\uparrow$ in 't'

$U = f(h)$ $\therefore U \propto \frac{1}{t}$

$h = f(t)$

depends on 't' implicitly through 'h'

# TISE :-

let
$$\psi_m = A e^{-i(\omega_m t - \vec{k}_m \cdot \vec{r}_m)} \quad \text{—— ①}$$

$i = \sqrt{-1}$, $\omega$ = angular freq.
$k$ = propagation const.

$$V_p = \frac{\omega}{k} \quad \text{—— ②}$$

$\psi$ must satisfy standard wave eqn.

$$\nabla^2\psi - \frac{1}{V_p^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad \text{—— ③}$$

4

Scanned with CamScanner

$$\frac{\partial \psi}{\partial t} = \left[ A e^{-i(\omega t - \vec{k}\cdot\vec{r})} \right] (-i\omega - 0)$$

$$\boxed{\frac{\partial \Psi}{\partial t} = -i\omega \Psi} \text{ — (4)}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega) \left( \frac{\partial \psi}{\partial t} \right)$$

$$\frac{\partial^2 \psi}{\partial t^2} = -i\omega (-i\omega \Psi)$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \Psi} \text{ — (5)}$$

from (5) & (3)

$$\nabla^2 \Psi + \frac{\omega^2}{V_p^2} \Psi = 0$$

$$\boxed{\nabla^2 \Psi + k^2 \Psi = 0} \text{ — (6)}$$

Dirac eqn.

Assume :- particle is travelling non-relativistically.

$$K.E. \approx \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$= \frac{\hbar^2 k^2}{2m}$$

$$E = K.E. + P.E.$$

$$= \frac{\hbar^2 k^2}{2m} + U$$

$$E - U = \frac{\hbar^2 k^2}{2m}$$

$$k^2 = \frac{2m}{\hbar^2}(E-U) \quad \text{——— ⑦}$$

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2}(E-U)\psi = 0}$$

3-D TISE for a restricted particle / particle subjected to some ext. force.

for a free particle $U = 0$

$$\therefore \boxed{\nabla^2 \psi + \frac{2mE}{\hbar^2}\psi = 0}$$

for 1-D

$$\nabla^2 \psi \longrightarrow \frac{d^2\psi}{dx^2}$$

# <u>TDSE</u>:-

$$\psi = A e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

Non relativistic motion is assumed.

$E = K.E. + P.E.$

$$E = \frac{p^2}{2m} + U$$

multiply by $\Psi$

$$E\Psi = \frac{p^2}{2m}\Psi + U\Psi \Rightarrow E\Psi = \frac{\hbar^2 k^2}{2m}\Psi + U\Psi$$

Take partial derivative of $\Psi$ w.r.t. '$t$'

$$\frac{\partial \Psi}{\partial t} = \left[A e^{-i(\omega t - \vec{k}.\vec{r})}\right](-i\omega)$$

$$= \Psi(-i\omega)$$

multiply both sides by $i\hbar$

$$i\hbar \frac{\partial \Psi}{\partial t} = i\hbar(\Psi)(-i\omega)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \hbar\omega\Psi$$

$$\therefore \quad i\hbar \frac{\partial \Psi}{\partial t} = E\Psi \quad \text{———} \quad 3$$

Take partial derivative of $\Psi$ w.r.t. '$x$'

$$\frac{\partial \Psi}{\partial x} = \left[A e^{-(\omega t - \vec{k}.\vec{r})}\right](+ik_x)$$

$$\frac{\partial \Psi}{\partial x} = ik_x\Psi \quad \text{———} \quad 4$$

Scanned with CamScanner

Similarly :-

$$\frac{\partial \psi}{\partial y} = i k_y \psi \quad , \frac{\partial \psi}{\partial z} = i k_z \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = i k_x \frac{\partial \psi}{\partial x} = i k_x (i k_x \psi)$$

$$\therefore \frac{\partial^2 \psi}{\partial x^2} = -k_x^2 \psi \quad \text{—— (5)}$$

Similarly :-

$$\frac{\partial^2 \psi}{\partial y^2} = -k_y^2 \psi \quad \text{—— (6)} \quad , \frac{\partial^2 \psi}{\partial z^2} = -k_z^2 \psi \quad \text{—— (7)}$$

$$\therefore \nabla^2 \psi = -k^2 \psi \quad \text{—— (8)}$$

$$\therefore E \psi = \frac{\hbar^2 k^2}{2m} \psi + U \psi$$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + U \psi}$$ ***

3-D TDSE for a restricted particle

for a free particle $U = 0$

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi}$$ **

for 1-D

$$\nabla^2 \psi \longrightarrow \frac{\partial^2 \psi}{\partial x^2}$$

# Limitations of Schrödinger eqn.:-

① Applicable only for non relativistic problems.

② 1st particle. 2nd particle - - - - - - - -
$(x_1, y_1, z_1), (x_2, y_2, z_2), - - - - - -$

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - U)\psi = 0$$

for 1st particle eqn :- $\nabla_1^2 \psi_1 + \frac{2m_1}{\hbar^2}(E_1 - U_1)\psi_1 = 0$

for 2nd particle eqn :- $\nabla_2^2 \psi_2 + \frac{2m_2}{\hbar^2}(E_2 - U_2)\psi_2 = 0$

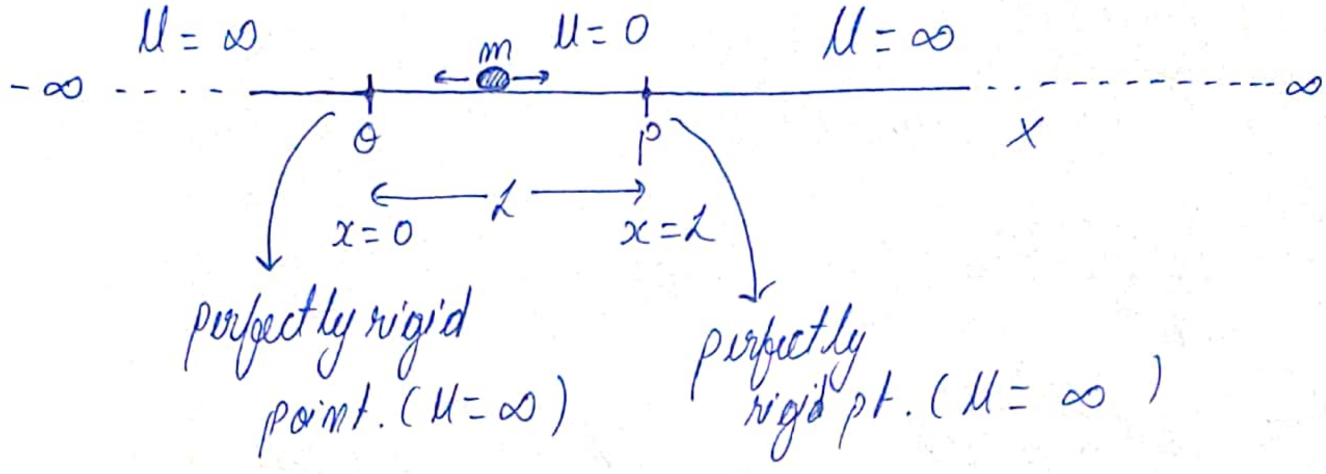
i.e. for many particle system this ~~so~~ ~~st~~ schrödinger eqn. is very difficult to solve

③ for solving schrödinger eqn. we require very high performance computational tools

④ schrödinger eqn. on solving gives us many solution out of which large no. of solutions are meaningless

⑤ schrodinger eqn. tells us ~~about~~ only about n, l and m but does not provide any result for 's' (spin q. no.)

# Particle in a one dimensional box :->

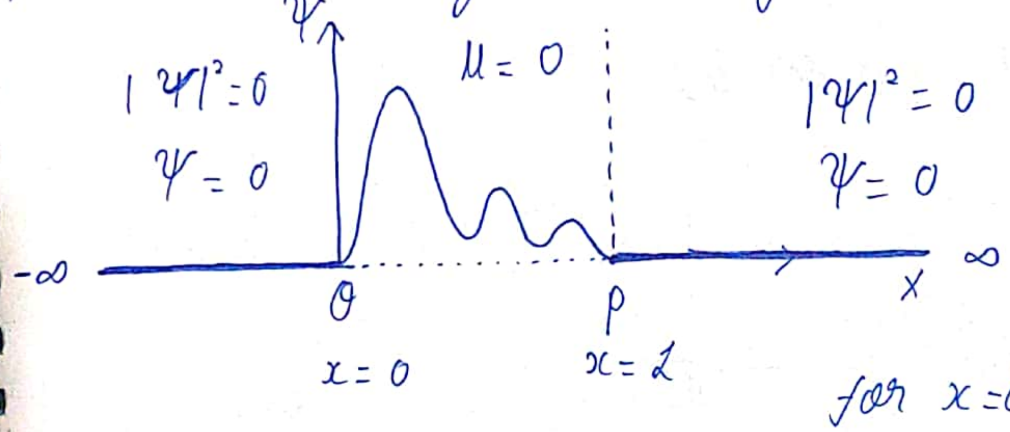



We know that every system tends to occupy the state of minimum potential energy

since, the particle is unable to cross 'O' and 'P' pt.

$\therefore$ In quantum mechanical way we can say that perfectly rigid pt. or perfectly rigid region must have infinite potential energy

Infinite potential energy gives unstability to particle so, particle will never go in that region




$$\left.\begin{array}{l} \text{for } x = 0,\ \psi = 0 \\ \text{for } x = l,\ \psi = 0 \end{array}\right\} - (1)$$

Assume b/w 'O' and 'P' particle is not interacting with anything.

$\therefore$ b/w 'O' and 'P' $U = 0$

$\therefore$ Total energy from O to P is kinetic energy

1-D TISE is given by:→

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

(But $U = 0$)

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad \text{—— ②}$$

Suppose $\frac{2mE}{\hbar^2} = k^2$ —— ③ ('k' is any const.)

$$\therefore \frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{—— ④}$$

soln. of ④ is of the form.

$a = -\omega^2 y$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

$y = A\sin(\omega t) + B\cos(\omega t)$

$$\psi = A\sin(kx) + B\cos(kx) \quad \text{—— ⑤}$$

for $x = 0$, $\psi = 0$

$\therefore 0 = A\sin(0) + B\cos(0)$

$\therefore B = 0$

$$\therefore \psi = A\sin(kx) \quad \text{—— ⑥}$$

for $x = L$, $\psi = 0$

$$0 = A\sin(kL) \quad \text{—— ⑦}$$

either $A = 0$ or $\sin(kL) = 0$

(non logical soln.)

Scanned with CamScanner

$\therefore$ if $A = 0$

$\therefore \Psi = 0$ (inside)

$\Psi = 0$ (outside) $\therefore$ particle is not present in the whole universe.

$\sin(kL) = 0$

$\therefore \boxed{kL = n\pi}$ —— ⑧ (quantum condition)
(stability ")

for $n = 0$ $\therefore kL = 0$
$A \neq 0$

$\Psi = A\sin\left(\frac{n\pi}{L}x\right)$ (rejected)

$\therefore \Psi = 0$

$n = -1$

$\underset{(+ve)(+ve)}{kL} = -\pi$ $\therefore$ (rejected) $n = -ve$ (rejected)

$\frac{2\pi}{\lambda}$

$\therefore n = 1, 2, 3, \ldots\ldots\ldots$

$k = \frac{n\pi}{L}$ , $k^2 = \frac{n^2\pi^2}{L^2}$

$\therefore \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$

$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$

$E = n^2(\varnothing)$ $\left[\varnothing = \frac{\pi^2\hbar^2}{2mL^2}\right]$

$\therefore \boxed{E_n = n^2(\varnothing)}$ ***

$E_1 = 1^2 \wp = \wp$ (ground state energy)

$E_2 = 2^2 \wp = 4\wp$ (1st excited state)

$E_3 = 3^2 \wp = 9\wp$ (2nd " ")

$E_4 = 4^2 \wp = 16\wp$ (3rd " ")

$\vdots \quad \vdots \quad \vdots \quad \vdots$

$\Psi = A \sin(kx)$

$\Psi = A \sin\left(\frac{n\pi}{L}x\right)$ — (10)

Assertion:- 'A' is chosen in such a way that $\Psi$ becomes normalized.

i.e. $|\Psi|^2$ = exactly probability density

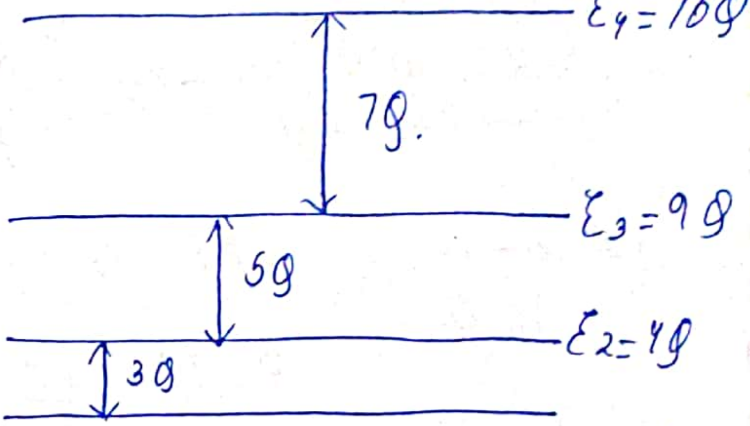
$= \frac{dP}{dx}$

$\Rightarrow \int_{x=0}^{x=L} dP = 1$

$\int_{x=0}^{x=L} |\Psi|^2 dx = 1$

$A^2 \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) dx = 1$

$\frac{A^2}{2} \int_0^L \left(1 - \cos\left(\frac{2n\pi x}{L}\right)\right) dx = 1$




$$\boxed{\wp = \frac{\pi^2 \hbar^2}{2mL^2}}$$

$U = 0$, $\therefore \boxed{E_n = K.E}$

$\therefore E_{min} = K.E_{min} = E_1 = \wp \neq 0$


never at rest.

$\frac{A^2}{2}[L - 0] = 1$

$\therefore A = \sqrt{\frac{2}{L}}$

$\therefore$ Normalized wave fn.

$$\boxed{\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)} \quad \text{—— (11)}$$

→ Eigen fn.

$$\boxed{E_n = n^2 g = \frac{n^2 \pi^2 \hbar^2}{2mL^2}} \quad \text{—— (12)}$$

→ Energy eigen value fn.

$E_n = g, 4g, 9g, 16g, 25g, 36g, \text{------}$

why not $1.5g, 2g, 3g \ldots$ ?? (not permissible levels)

$$\left[\because L = \frac{nh}{2\pi} = n\hbar\right]$$

$k = \frac{n\pi}{L}$ , $p = \hbar k$

$$p = \hbar\left(\frac{n\pi}{L}\right)$$

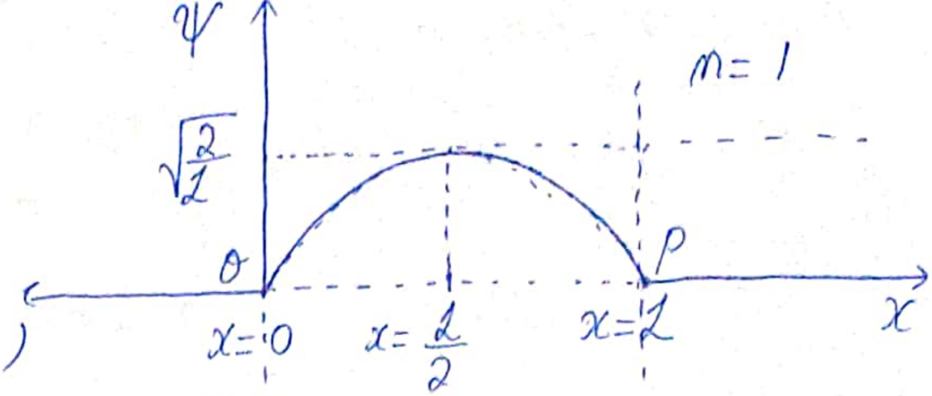
$$p = \frac{h}{2\pi}\left(\frac{n\pi}{L}\right)$$

$$\boxed{p = \frac{nh}{2L}} \quad \text{—— (13)} \qquad \boxed{L = \frac{nh}{2\pi}}$$

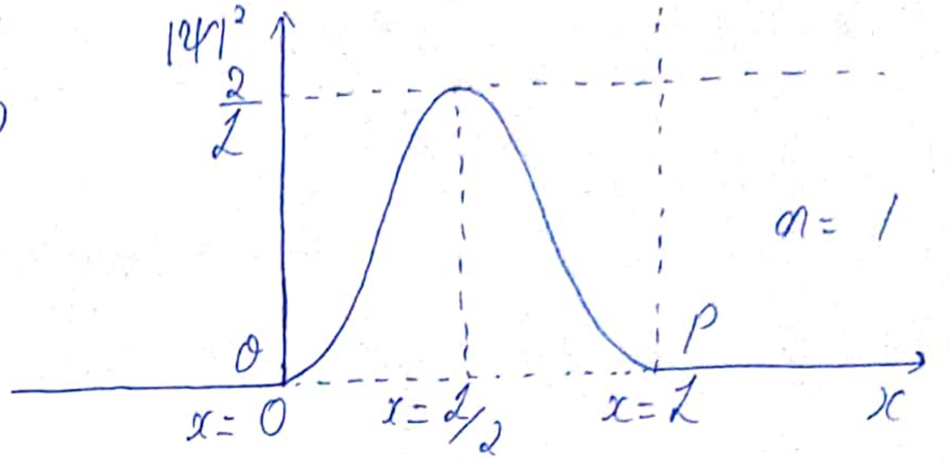
$$\psi = \sqrt{\frac{2}{L}}\, \sin\left(\frac{n\pi}{L} x\right)$$

$$\psi = \sqrt{\frac{2}{L}}\, \sin\left(\frac{\pi}{L} x\right)$$

(for $n=1$)

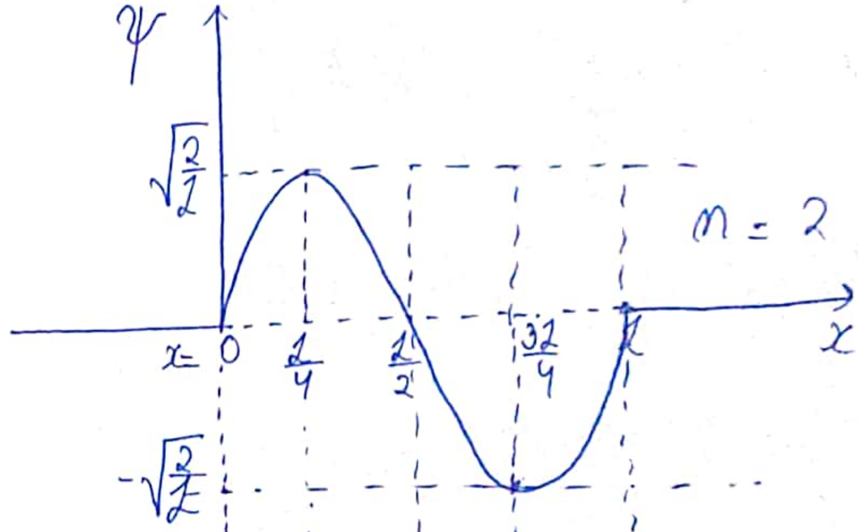



| $x=0$ | $x=\frac{L}{2}$ | $x=L$ |
|---|---|---|
| $\psi=0$ | $\psi=\sqrt{\frac{2}{L}}$ | $\psi=0$ |



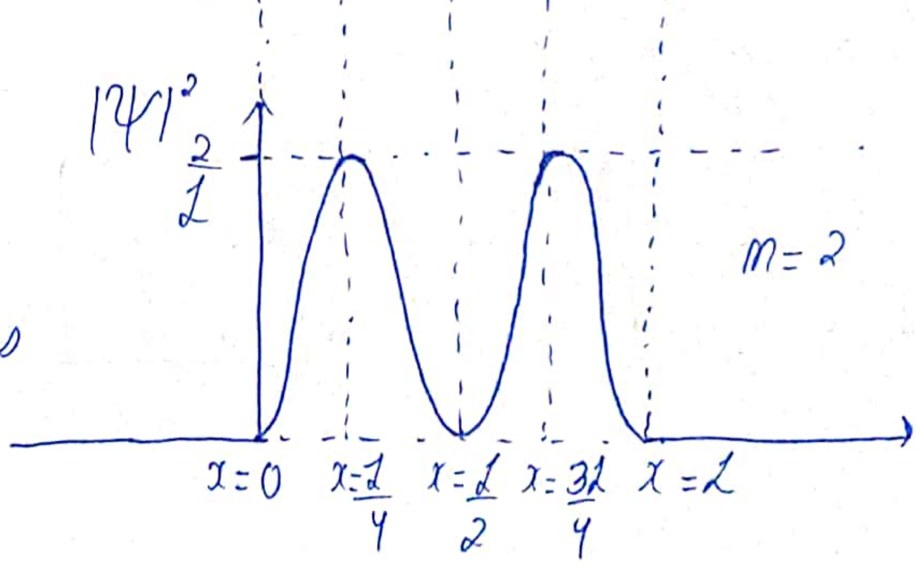

for $n = 2$

$$\psi = \sqrt{\frac{2}{L}}\, \sin\left(\frac{2\pi x}{L}\right)$$




| $x=0$ | $x=\frac{L}{4}$ | $x=\frac{L}{2}$ |
|---|---|---|
| $\psi=0$ | $\psi=\sqrt{\frac{2}{L}}$ | $\psi=0$ |

| $x=\frac{3L}{4}$ | $x=L$ |
|---|---|
| $\psi=\sqrt{\frac{2}{L}}$ | $\psi=0$ |




for $n = n$ $\longrightarrow$ $n$-antinodes

$\longrightarrow$ $n+1$ nodes

Ques:- What is the wavelength of de-Broglie wave attached with an $e^-$ having speed (a) 300 m/sec (b) $3\times10^7$ m/sec.

Soln. We know $m_0 = 9.1\times10^{-31}$ kg.

(i) $V = 300$ m/sec. $V <<< c$

$$\therefore \lambda = \frac{h}{mV} = \frac{h}{\frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}}} \approx \frac{h}{m_0 V} \quad \left(\because \frac{V^2}{c^2} \approx 0\right)$$

as $V <<< c$

$$= \frac{6.63\times10^{-34}\text{ J sec.}}{9.1\times10^{-31}\text{ kg}\times300\text{ m/sec.}} = 2.42\times10^{-3}\times10^{-34}\times10^{+31}$$

$$= 2.42\times10^{-6}\text{ m}$$

(ii) $V = 3\times10^7$ m/sec. $\longrightarrow$ comparable with c

$\therefore m \neq m_0$

$$m = \frac{m_0}{\sqrt{1-\frac{\cancel{9}\times10^{14}}{\cancel{9}\times10^{16}}}}$$

$$m = \frac{m_0}{\sqrt{1-0.01}} = \frac{9.1\times10^{-31}}{0.99} = 9.2\times10^{-31}\text{ kg.}$$

$$\lambda = \frac{h}{m_0 V} = \frac{6.63\times10^{-34}}{9.2\times10^{-31}\times3\times10^7} = 0.24\times10^{-34+31-7}\text{ m}$$

$$\lambda = 0.24\times10^{-10}\text{ m}$$

To find ~~velocity~~ wavelength of electron with 80% speed of light.

sol$^n$:- Given $V = 80\% \ c$

$$= \frac{80}{100} \times 3 \times 10^8 = 0.8\,c$$

$$m = \frac{m_0}{\sqrt{1 - \frac{64\,\cancel{c^2}}{100\,\cancel{c^2}}}} = \frac{m_0}{\sqrt{1-0.64}} = \frac{m_0}{\sqrt{0.36}}$$

$$m = \frac{m_0}{0.6} = \frac{9.1 \times 10^{-31}}{0.6} = 15.17 \times 10^{-31}\ kg.$$

$$\lambda = \frac{h}{mV} = \frac{6.63 \times 10^{-34}}{15.17 \times 10^{-31} \times 0.8 \times 3 \times 10^{8}}$$

$$\lambda = 0.182 \times 10^{-11}$$

Scanned with CamScanner