Question Bank (Multiple Integrals)

Q1 Evaluate $\iint_S \sqrt{xy-y^2}\,dx\,dy$, where S is the triangle with vertices (0,0), (10,1) and (1,1)

Q2. Evaluate $\iint_R y\,dx\,dy$, where R is the region bounded by the parabolas $y^2=4x$ and $x^2=4y$

Q3. Sketch the region of integration and Evaluate the following integrals

$$\int_1^{\log 8}\int_0^{\log y} e^{x+y}\,dx\,dy$$

Q4. Sketch the region of integration and Evaluate the following integrals

$$\int_0^2\int_0^{y^2} e^{x/y}\,dx\,dy$$

Q5 Sketch the region of integration and Evaluate $\iint_R (y-2x^2)\,dx\,dy$ where R is the region inside the square $|x|+|y|=1$

Q6 Evaluate $\iint r^3\,dr\,d\theta$, over the area bounded between the circles $r=2\cos\theta$ and $r=4\cos\theta$

Q7. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circles $x^2+y^2=a^2$

Q8. Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2=a^2\cos 2\theta$

Q9. Show that $\iint_R r^2 \sin\theta \, dr\, d\theta = \frac{2a^3}{3}$

where R is the region bounded by the semi-circle

$r = 2a\cos\theta$, above the initial line

Q10. Evaluate $\iint r\sin\theta \, dr\, d\theta$ over the area of the cardiod $r = a(1+\cos\theta)$ above the initial line.

Q11 Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy\, dx$ and hence evaluate the same.

Q12. Evaluate the changing the order of integration of $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x \, dx\, dy}{\sqrt{x^2+y^2}}$

Q13. Change the order of integration of $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V \, dy\, dx$

Q14. Change the order of integration and evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\, dx$

Q15 Change the order of integration for $\int_0^a \int_{mx}^{lx} f(x,y)\, dy\, dx$

Q16 Evaluate by changing the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy\, dx$

Q17. Evaluate by changing the order of integration $\int_0^\infty \int_0^x x e^{-x^2/y} \, dy\, dx$

18. Evaluate the following integrals by changing the order of Integration

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)\, dx\, dy \quad ; a>0$$

Q19. Evaluate by changing the order of Integration

(a) $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xy \, dx\, dy$ (b) $\int_0^1 \int_x^1 \sin y^2 \, dy\, dx$

Q20. Evaluate $\int_0^2 \int_1^2 \int_0^{yz} xyz \, dx\, dy\, dz$

Q21. $\int_{-1}^1 \int_0^z \int_{x+z}^{x+z} (x+y+z)\, dy\, dx\, dz$

Q22 $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz\, dy\, dx$

Q23. $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r \, dz\, dr\, d\theta$

Q24. Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the tetrahedron bounded by the coordinates planes and the plane $x+y+z=1$

Q25 $\int_0^a \int_0^a \int_0^a (yz+zx+xy)\, dx\, dy\, dz$

Q26. Evaluate $\iint \sqrt{a^2-x^2-y^2}\, dx\,dy$ over the circle $x^2+y^2=ax$ in the positive quadrant

Q27. Evaluate the following by changing to polar coordinates -

(i) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\, dy\, dx}{\sqrt{x^2+y^2}}$ (ii) $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \log(x^2+y^2+1)\, dx\,dy$

Q28. Evaluate $\iiint \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$, the integral being extended to the positive octant of the sphere $x^2+y^2+z^2=1$

Q29. Using the transformation $x+y=u$, $y=uv$ show that $\iint \sqrt{xy(1-x-y)}\, dx\,dy = \frac{2\pi}{105}$, integration being taken over the area of the triangle bounded by the lines $x=0$, $y=0$ and $x+y=1$

Q30. Evaluate the following by changing into polar - coordinates

$$\int_0^{4a} \int_{y^2/4a}^{y} \frac{x^2-y^2}{x^2+y^2}\, dx\,dy$$

Q31. Find the area lying between $y=4x-x^2$ and line $y=x$

Q32. Find the area bdd by $x=2y-y^2$ and $x=y^2$

Q33. Find the area between the parabolas $y^2=4ax$, $x^2=4ay$

Q34. find the area of region bounded by the lines $x=-2$, $x=2$ and Circle $x^2+y^2=9$

Q35 find the smaller area bounded by the ellipse $4x^2+9y^2=36$ and the straight line $2x+3y=6$

Q36. find area lying inside the Circle $r=a\sin\theta$ and outside the Cardiod $r=a(1-\cos\theta)$

Q37. find the area outside the circle $r=a$ and inside the cardiod $r=a(1+\cos\theta)$

Q38. find area inside the lemmiscate $r^2=a^2\cos 2\theta$

Q39. find the Volume enclosed between tetrahedron bounded by the coordinate planes & plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

Q40. find the Volume bounded by the cylinder $x^2+y^2=4$ and planes $y+z=4$ and $z=0$

Q41. find the Volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$

Q42 find the area bounded by the parabolas $y^2=4-x$ and $y^2=4-4x$.

Q43. find by triple integration, the Volume of the paraboloid of revolution $x^2+y^2=4z$ cut off by the plane $z=4$

Q 44. Find the volume of sphere of radius a by triple integration.

Q45 Find by triple Integration, the Volume bdd by the paraboloid $az = x^2 + y^2$ and the cylinder $x^2 + y^2 = R^2$

Q46. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$

Q47. Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$

Q48. Evaluate by changing the order of Integration

$$\int_0^\infty \int_0^\infty \frac{e^{-y}}{y}\, dy\, dx$$

Q49. Find the area enclosed by the curves $y^2 = x^3$ and $y = x$

Q50. Change into polar coordinates $\int_0^1 \int_x^{\sqrt{2x-x^2}} f(x,y)\, dy\, dx$

Question Bank (Vector Calculus)

Q1. Find the unit vector normal to the surface $z = x^3 + y^2$ at $(1,-2,5)$ and $z = \sqrt{x^2+y^2}$ at $(3,4,5)$

Q2. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
Show that (i) $\text{grad } r = \dfrac{\vec{r}}{r}$

(ii) $\text{grad}\left(\dfrac{1}{r}\right) = \dfrac{-\vec{r}}{r^3}$

(iii) $\nabla r^n = n r^{n-2}\vec{r}$

Q3. If $r = |\vec{r}|$, Prove that

(i) $\nabla(\log r) = \dfrac{\vec{r}}{r^2}$

(ii) $\text{grad } |\vec{r}|^2 = 2\vec{r}$

Q4. what is the directional derivative of $2xy + z^2$ at the point $(1,-1,3)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$?

Q5. find the maximum value of the directional derivative of $f = x^2 - 2y^2 + 4z^2$ at the point $(1,1,-1)$

Q6. What is the directional derivative of the function $xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of the normal to the surface $x \log z - y^2 = 4$ at $(-1,2,1)$

Q7. Prove (i) div $(\phi \vec{A}) = \phi\,(\text{div}\,\vec{A}) + (\text{grad}\,\phi)\cdot\vec{A}$

(ii) Curl $(\phi \vec{A}) = (\text{grad}\,\phi) \times \vec{A} + \phi\,\text{curl}\,\vec{A}$

(iii) Curl $(\text{grad}\,\phi) = \nabla \times (\nabla \phi) = \vec{0}$

(iv) Div $(\text{grad}\,\phi) = \nabla^2 \phi$

(v) Div $(\text{Curl}\,\vec{V}) = \nabla \cdot (\nabla \times \vec{V}) = 0$

(vi) Curl $(\text{Curl}\,\vec{V}) = \text{grad div}\,\vec{V} - \nabla^2 \vec{V}$

Q8. If $u = x^2 + y^2 + z^2$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then find div $(u\vec{r})$ in terms of u

Q9. Prove that (i) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$

(ii) $\nabla^2 (r^n) = n(n+1)\, r^{n-2}$

Q10. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, Prove that div $\vec{r} = 3$ and Curl $\vec{r} = \vec{0}$

Q11. If $\vec{F} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$, show that $\vec{F} \cdot \text{Curl}\,\vec{F} = 0$

Q12. find $\nabla \times (\nabla \phi)$ if $\phi(x,y,z) = -2x^3 y z^2$

Q13. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Prove that $\nabla^2 \left(\frac{1}{r}\right) = 0$

Q14. A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$

Evaluate the line integral over circular path given by $x^2 + y^2 = a^2$ and $z = 0$

Q15 Compute $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F} = \dfrac{\hat{i}y - \hat{j}x}{x^2+y^2}$

and C is the circle $x^2+y^2=1$.

Q16 Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz-y)\hat{j} + 3\hat{k}$ along a straight line from $(0,0,0)$ to $(2,1,3)$

Q17. Show that the circulation of $\vec{F}$ round the curve C, where $\vec{F} = e^x \sin y\,\hat{i} - e^x \cos y\,\hat{j}$ and C is the rectangle whose vertices are $(0,0)$, $(1,0)$, $(1,\frac{\pi}{2})$, $(0,\frac{\pi}{2})$ is Zero.

Q18. Compute the line integration $\int_C y^2dx - x^2dy$ along the triangle whose vertices are $(1,0)$, $(0,1)$ & $(-1,0)$

Q19 Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = y\hat{i} + 3\hat{j} + x\hat{k}$ & where C is the circle $x^2+y^2=1$, $z=0$

Q20. Show that the surface integration of $\iint_S \vec{F}\cdot\hat{n}\,ds = \frac{3}{2}$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and the surface S is the cube bdd by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$

Q21. Compute $\iint_S \vec{F}\cdot\hat{N}\,ds$, $\vec{F} = 6z\hat{i} - 4\hat{j} + y\hat{k}$ and S is the portion of plane $2x+3y+6z=12$ in first octant

Q22. Evaluate $\iint_S \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = 2x^2y\,\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$

and S is the closed surface of the region in the first Octant bounded by Cylinder $y^2+z^2=9$ & plane $x=0$, $x=2$, $y=0$, $z=0$

Q23 Evaluate $\iint_S \vec{F}\cdot d\vec{s}$ where $\vec{F} = x\hat{i} + (z^2-zx)\hat{j} - xy\hat{k}$

and S is the triangular surface with Vertices $(2,0,0)$, $(0,2,0)$ & $(0,0,4)$

Q24, Define gradient of a Scalar point function and find gradient of y^2-4xy at $(1,2)$

Q25 Define Divergence of vector pt function. and find divergence of $\vec{F}$ where $\vec{F} = \text{grad}\,(x^3+y^3+z^3-3xyz)$

Q26. Define Curl of a Vector pt function and Prove that Curl Curl $\vec{V}$ = grad div $\vec{V} - \nabla^2\vec{V}$

Q27. Evaluate $\iint_S (yz\,\hat{i} + 3x\hat{j} + xy\hat{k})\cdot\hat{n}\,ds$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant.

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Curve Tracing

Q:-1 Trace the following curves:

(i) $x^3 + y^3 = 3axy$; $a > 0$

(ii) $a^2y^2 = x^2(a^2 - x^2)$

(iii) $x(x^2+y^2) = a(3x^2 - y^2)$

(iv) $y = x + \frac{1}{x}$

(v) $y^2(a-x) = x^3$; $a > 0$

(vi) $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ or $x = a\cos^3\theta$, $y = b\sin^3\theta$.

(vii) $y = c\cosh\frac{x}{c}$.

Q:-2 Trace the following Polar curves:

(i) $r = a(1-\cos\theta)$, $r = a(1-\sin\theta)$
$r = a(1+\cos\theta)$ $r = a(1+\sin\theta)$

(ii) $r = a$, $r = 2a\cos\theta$, $r = 2a\sin\theta$

(iii) $r = a\sin 3\theta$ (iv) $r = a + b\cos\theta$ $(a > b)$

(v) $r^2 = a^2\cos 2\theta$ (vi) $r = a\cos 3\theta$.