Electromagnetic waves

Important definitions :-

1 <u>Scalar field</u> → The region in space, in which a scalar quantity has unique value or magnitude at every point is called scalar quantity. For exp → Temperature and electric potential

2 <u>Vector field</u> → The region in space in which a vector quantity has unique value (magnitude and direction) at every point is called vector field. For exp → Electric field, gravitational field etc

3 <u>Gradient</u> → Gradient of a scalar field function is defined as operation of $\vec{\nabla}$ on scalar field function then its gradient is given by

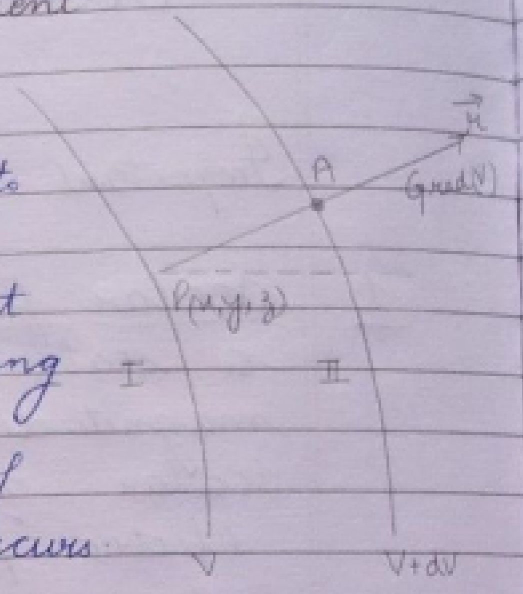
$$\text{grad}(V) = \vec{\nabla} V$$

4 We define differential operator

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

This operator is called '<u>Del</u>' operator

Physical significance of Gradient



The gradient of a scalar field at a point is equal to maximum rate of change of function w.r.t position of that point and is directed along the direction in which this maximum rate of change of scalar field function occurs.

Drawn figure shows a scalar field function $V(x,y,z)$ having two equifield surfaces. Thus value of function everywhere on surface I is V and for all points on surface II is $V+dV$.

If we move from point P on surface I to points A or B on surface II, then change in V in both cases is dV.

However distance PA is least. Hence rate of change of V is $\left(\frac{dV}{dr}\right)$ which is maximum along $\hat{r}$ (a unit vector along PA)

Thus grad $V = \left(\frac{dV}{dr}\right)_{max} \hat{r}$

It can be shown that grad V is given by

grad $V = \vec{\nabla} V$

Note that although V is a scalar function, however its gradient is a vector quantity this fact is easily demonstrated. There we have actually found gradient of scalar field function $V(x, y, z)$.

The divergence of a vector field at a point is defined as its dot product with $\vec{\nabla}$ i.e if $\vec{E}$ is any vector, then its divergence at a point is given as,

$$\text{div}(\vec{E}) = \vec{\nabla} \cdot \vec{E}$$

Divergence of a Vector field

The divergence of a vector field at any point gives maximum flux (of the vector field) per unit volume of a closed surface drawn around that point such that volume of closed surface approaches zero.

If $\vec{F}$ represents a vector field, i.e $\vec{F}$ be a continuously differentiable vector point function, the function.

$$\hat{i} \cdot \frac{\partial \vec{F}}{\partial x} + \hat{j} \cdot \frac{\partial \vec{F}}{\partial y} + \hat{k} \cdot \frac{\partial \vec{F}}{\partial z}$$

is a scalar and it is called divergence of $\vec{F}$. It is written as div $\vec{F}$.

Now $\operatorname{div} \vec{F} = \hat{i} \cdot \frac{\partial \vec{F}}{\partial x} + \hat{j} \cdot \frac{\partial \vec{F}}{\partial y} + \hat{k} \cdot \frac{\partial \vec{F}}{\partial z}$

$$= \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \cdot \vec{F}$$

$$= \nabla \cdot \vec{F}$$

So that divergence of a vector point function $\vec{F}$ is the scalar product of the Del operator with $\vec{F}$.

Curl of a Vector field

The magnitude of curl of a vector field at a point gives maximum circulation per unit area of a closed path drawn around that point such that area of closed path approaches zero.

The function $\hat{i} \times \frac{\partial \vec{F}}{\partial x} + \hat{j} \times \frac{\partial \vec{F}}{\partial y} + \hat{k} \times \frac{\partial \vec{F}}{\partial z}$ is a vector and it is called curl of $\vec{F}$. It is written as curl of $\vec{F}$.

$$\operatorname{Curl} \vec{F} = \hat{i} \times \frac{\partial \vec{F}}{\partial x} + \hat{j} \times \frac{\partial \vec{F}}{\partial y} + \hat{k} \times \frac{\partial \vec{F}}{\partial z}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \vec{F}$$

$$= \nabla \times \vec{F}$$

i.e curl of a vector point function $\vec{F}$ is the vector product of the Del operator with $\vec{F}$.

Solenoidal vector point function → A vector point function $\vec{F}$ is said to be solenoidal in a region if its flux across any closed surface in that region be zero. This happens when

$$\boxed{\text{div } \vec{F} = 0} \quad \text{or} \quad \boxed{\nabla \cdot \vec{F} = 0}$$

This means that for a vector point function to be solenoidal, either the lines of flow of its flux should form closed curves or extend to infinity.

Irrotational vector point function → A vector point function F is said to be irrotational if the curl of $\vec{F}$ is zero, i.e

$$\boxed{\text{curl } \vec{F} = 0} \quad \text{or} \quad \boxed{\nabla \times \vec{F} = 0}$$

Equation of continuity → The divergence of current density $\vec{J}$ at a point is numerically equal to the quantity of electric charge flowing out per second per unit volume (ρ) through an infinitesimal closed surface surrounding that point div J at a point is equal to the rate of decrease of charge density in the neighbourhood of that point

$$\nabla\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$

$$\boxed{\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

This is known as equation of continuity.

Some useful results

$$\text{div grad } \phi = \nabla\cdot\nabla\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \nabla^2$$

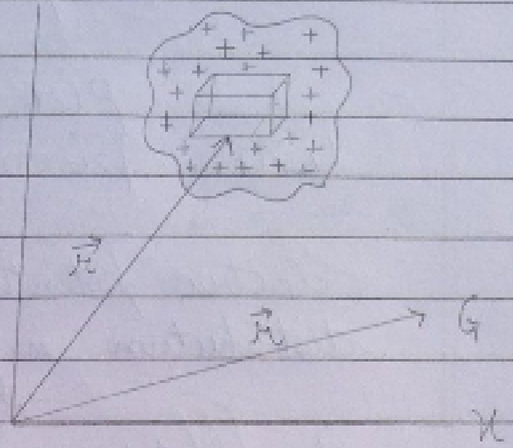
where ∇^2 is referred as the Laplacian operator

Curl grad $\phi = \nabla\times\nabla\phi = 0$, because the cross product of two equal vectors is zero

div curl $f = \nabla\cdot\nabla\times f = 0$, because it is a combination similar to $A\cdot A\times B$ where $A\times B$ is perpendicular to A and has no component in the direction of A

curl curl $f = \nabla\times(\nabla\times f) = \nabla\nabla\cdot f - \nabla^2 f$ = grad div $f - \nabla^2 f$

Relation between electric field and electric potential at a point



Draw figure shows a volume charge distribution Let A (x', y', z') is any point in the distribution having position vector $\vec{r}'$. Thus,

$$\vec{r}' = x\hat{i} + y\hat{j} + z\hat{k}$$

Draw a small volume $dv' = dx'dy'dz'$ around point A. Let charge enclosed in elementary volume dv' is dq.

Suppose $\rho(x', y', z') \equiv \rho(\vec{r}')$ is volume charge density at point A. Notice that ρ need not be uniform everywhere in the distribution, so it is a function of position of point A in the distribution.

Consider an observation point $G(x, y, z)$ in vacuum having position vector $\vec{r}$. Thus,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Electric potential at point G due to elementary volume dv' is given as

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{dq}{|\vec{r} - \vec{r}'|}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{r}')\,dv'}{|\vec{r} - \vec{r}'|} \qquad \because \rho(\vec{r}') = \frac{dq}{dv'}$$

Electric potential at point G due to complete distribution is given as

$$V = \iiint dV = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\vec{r}')\,dv'}{|\vec{r} - \vec{r}'|} \quad \text{——— (1)}$$

Now the electric field at a point G due to volume element dv' is given as

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{dq(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

Total electric field at G due to complete distribution is given as

$$\vec{E} = \iiint d\vec{E} = \frac{1}{4\pi\varepsilon_0} \iiint \rho(\vec{r}') \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dv'$$

$$= \frac{-1}{4\pi\varepsilon_0} \iiint \rho(\vec{r}') \left\{ \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r}'|} \right) \right\} dv' \qquad \because \vec{\nabla}\left(\frac{1}{|\vec{r} - \vec{r}'|}\right) = \frac{-(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$= -\vec{\nabla}\left\{\frac{1}{4\pi\varepsilon_0}\iiint\frac{\rho\,\vec{r}'}{|\vec{r}-\vec{r}'|}\right\}dv'$$

Using eqⁿ ①

$$\vec{E} = -\vec{\nabla}V$$

Thus electric field at a point is equal to negative of gradient of potential.

Solid Angle

A solid angle is a 3D angular volume that is defined analogously to the definition of a plane angle in two dimensions. A solid angle ω, made up of all the lines from a closed curve meeting at a vertex, is defined by the surface area of sphere substended by the lines and by the radius of that sphere.

Stoke's theorem

It states that line integration of a vector field function around a closed path is equal to surface integration of the curl of vector field taken over an open surface bounded by the closed path. $\oint_C \vec{E}\cdot\vec{dl} = \iint_S(\vec{\nabla}\times\vec{E})\cdot d\vec{S}$

Or

This theorem states that the flux of the curl of a vector field $\vec{F}$ over any closed surface S of any shape is equal to the line integral of the vector field $\vec{F}$ over the boundary of that surface i.e

$$\iint_S \text{curl } \vec{F} \cdot d\vec{s} = \int \vec{F} \cdot \vec{dl}$$

Here ds represents infinitesimal element of area of surface S and $\vec{dl}$ and $\vec{ds}$ infinitesimal element of boundary C of the surface

Gauss divergence theorem

It states that the surface integration of a vector field function over a closed surface is equal to volume integration of the divergence of the function taken over the volume of the closed surface. $\oint_S \vec{E} \cdot \vec{ds} = \iiint_V (\nabla \cdot \vec{E}) dV$

or This theorem states that the flux of a vector field $\vec{F}$ over any closed surface S is equal to the volume integral of the divergence of the vector field over the volume enclosed by the surface S i.e

$$\iint_S \vec{F} \cdot d\vec{s} = \iiint_V \text{div } \vec{F} \, d\vec{V} = \iiint_V (\nabla \cdot \vec{F}) dV$$

~~Here~~ Let us consider, finite volume V of any shape enclosed by surface S.

1D = Circulational
2D = Curl

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In short

Gauss divergence theorem

$$\boxed{\oint_S \vec{A}\cdot\vec{ds} = \iiint_V (\vec{\nabla}\cdot\vec{A})\, dV}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\oint \vec{A}\cdot\vec{ds}}{dV}$$

$$\iint_V (\vec{\nabla}\cdot\vec{A})\, dV = \oint_S \vec{A}\cdot\vec{ds}$$ (through small element)

through full volume

Stoke theorem (curl of $\vec{A}$)

$$\vec{\nabla}\times\vec{A} = \frac{\oint \vec{A}\cdot\vec{dl}}{ds}\,\hat{n}$$

$$\iint_S (\vec{\nabla}\times\vec{A})\cdot\vec{ds} = \oint_C \vec{A}\cdot\vec{ds}$$

$$\boxed{\oint_C \vec{A}\cdot\vec{dl} = \iint_S (\vec{\nabla}\times\vec{A})\cdot\vec{ds}}$$

(If A is zero, then action is irrotational
If A is non zero then action is rotational)

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Maxwell's Equations

The electromagnetic equations are known as Maxwell's equations. These equations are actually the basic laws of electricity and magnetism whose integral and differential forms are as follows :-

(i) Maxwell's first equation

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Integrating on both sides over the open area of coil, we get

$$\iint \vec{\nabla} \times \vec{E} \cdot \vec{ds} = -\frac{\partial \vec{B}}{\partial t} \iint \vec{B} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \phi_B}{\partial t}$$ (Using stoke theorem)

(ii) Maxwell's Second equation

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integrating both sides over open area of closed path, we get

$$\iint \vec{\nabla} \times \vec{H} \cdot \vec{ds} = \iint \vec{J} \cdot \vec{ds} + \frac{\partial}{\partial t} \iint \vec{D} \cdot \vec{ds}$$

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$$\oint \vec{H} \cdot \vec{dl} = I + I_d \quad \text{(Using stoke theorem)}$$

$$I = \iint \vec{J} \cdot \vec{ds} = \text{conduction current}$$

$$I_d = \frac{\partial}{\partial t} \iint \vec{D} \cdot \vec{ds} = \text{displacement current}$$

(iii) Maxwell's third equation

$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

Integrating both sides over volume occupied by closed surface, we get

$$\iiint \vec{\nabla} \cdot \vec{D}\, dV = \iiint \rho_f\, dV$$

$$\oint\!\!\oint \vec{D} \cdot \vec{ds} = q_f \quad \text{(Using Gauss divergence theorem)}$$

(iv) Maxwell's fourth equation

$$\vec{\nabla} \cdot \vec{B} = 0$$

Integrating both sides over the volume occupied by closed surface, we get

$$\iiint \vec{\nabla} \cdot \vec{B}\, dV = 0$$

$$\oint\!\!\oint \vec{B} \cdot \vec{ds} = 0 \quad \text{(Using Gauss divergence theorem)}$$

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Importance of Maxwell's equations

Maxwell's first equation:-

(i) It relates electric field vector and magnetic flux density vector

(ii) It is time dependent equation.

(iii) It describes Faradgy's laws of electromagnetic induction along with Lenz's law.

(iv) It states that time varying magnetic field is a source of electric field in space

Maxwell's second equation:-

(i) It relates magnetic field vector with displacement vector.

(ii) It is time independent equation.

(iii) It summarizes Ampere's law for varrying currents

(iv) It tells that electric current $(\vec{J})$ is a source of magnetic field, which agrees well will Oersted's experiment.

(v) It tells that time varying electric field $(\because D = \epsilon\vec{E})$ is a source of magnetic field in space.

Maxwell's third equation:-

(i) It summarizes Gauss law.

(ii) It is a time independent or stedy state equation.

(iii) $\vec{\nabla}\cdot\vec{E}$ can be visualised as derivative of E. If ρ_f is +ve then $\vec{\nabla}\cdot\vec{E}$ is also +ve. It means that +ve charge is a source of electric field lines. Similarly -ve charge is a sink for electric field lines.

(iv) Since $\vec{\nabla}\cdot\vec{D}$ is always a scalar quantity. So this relation shows that charge density and hence charge is a scalar quantity.

Maxwell's fourth equation

(i) It tells that isolated magnetic monopoles do not exist.

(ii) It is a time independent or stedy state equation.

(iii) It tells that there are no sources or sinks for magnetic field lines i.e magnetic field lines form closed continuous loops.

Electromagnetic waves (EM waves)

Electromagnetic waves are coupled electric and magnetic oscillations that move with speed of light and exhibit typical wave behaviour.

⇒ The following properties are associated with electromagnetic wave through free space

(i) Electromagnetic waves travel with speed of light.

(ii) Electromagnetic waves are transverse in nature.

(iii) The ratio of electric to magnetic field $\left(\frac{\vec{E}}{\vec{B}}\right)$ in an electromagnetic wave equals to speed of light.

(iv) EM waves carry both energy and momentum, which can be delivered to a surface

Wave equation in free space

Consider the case of electromagnetic phenomena in free space or more generally, in a perfect dielectric containing no charges ($\rho = 0$) and no conduction currents ($J = 0$). For this case the field equations become,

$$\nabla \cdot \vec{D} = 0 \quad \text{---} \quad (1)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{---} \quad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---} \quad (3)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \text{---} \quad (4)$$

For free space $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$ and $\rho = 0$

$$\vec{D} = \varepsilon_0 \vec{E} \text{ and } \vec{B} = \mu_0 \vec{H}$$

Taking curl of eq^n (3) on both sides, we have

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t} (\nabla \times \vec{B})$$

$$= -\mu_0 \nabla \times \frac{\partial \vec{H}}{\partial t} \quad \text{---} \quad (5)$$

And from eq^n (4),

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

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$$\nabla \times \frac{\partial H}{\partial t} = \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{——— (6)}$$

$$\therefore \nabla \times (\nabla \times \vec{E}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla(\nabla \times \vec{E}) = \nabla \nabla . \vec{E} - \nabla^2 \vec{E} \text{ and } \nabla \cdot \vec{E} = \frac{1}{\varepsilon_0} \nabla . \vec{D} = 0$$

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{——— (7)}$$

Similarly we can solve for magnetic field

$$\boxed{\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{——— (8)}$$

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Ques Show that EM waves are transverse in nature.

Ans From Maxwell's first equation, we have

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \quad \text{—— (1)}$$

$$\vec{E}(\vec{r}, t) = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\hat{i} E_x + \hat{j} E_y + \hat{k} E_z = (\hat{i} E_{0x} + \hat{j} E_{0y} + \hat{k} E_{0z})\, e^{i(\vec{k}\cdot\vec{r}-\omega t} \quad \text{—— (2)}$$

Comparing both sides of above eq^n

$$\left[\begin{aligned} k\cdot r &= (\hat{i}k_x + \hat{j}k_y + k_z)\cdot(\hat{i}x + \hat{j}y + \hat{k}z) \\ &= k_x x + k_y y + k_z z \end{aligned}\right]$$

$$E_x = E_{0x}\, e^{i(k\cdot r-\omega t)}$$

$$= E_{0x}\, e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$\frac{\partial E_x}{\partial x} = E_{0x}\, e^{i(k_x x + k_y y + k_z z - \omega t)} \times i k_x$$

$$\frac{\partial E_x}{\partial x} = i k_x E_x \quad \text{—— (2)}$$

Similarly $\frac{\partial E_y}{\partial y} = i k_y E_y$ —— (3)

$$\frac{\partial E_z}{\partial z} = i k_z E_z \quad \text{—— (4)}$$

Put (2) (3) (4) in (1)

$$i[k_x E_x + k_y E_y + k_z E_z] = 0$$

$i(\vec{K}\cdot\vec{E}) = 0$ $\theta = 90^\circ$

$\vec{K}\cdot\vec{E} = 0$

$\vec{K} \perp \vec{E}$

Similarly

$\vec{K}\cdot\vec{B} = 0$

$\vec{K} \perp \vec{B}$

Now from Maxwell's eq^n

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{—— (5)}$$

$$\vec{B} = \vec{B_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\frac{\partial\vec{B}}{\partial t} = \vec{B_0}\, e^{(\vec{k}\cdot\vec{r}-\omega t)}(-i\omega)$$

$$= -i\omega\vec{B} \quad \text{—— (6)}$$

Using (6) in (5)

$$i\vec{K}\times\vec{E} = i\omega\vec{B}$$

$$\cancel{\vec{K}\times B} \quad \vec{K}\times\vec{E} = \omega\vec{B}$$

$$= \vec{B}\perp\vec{E}$$

$$\vec{\nabla}\times\vec{E} = i\vec{K}\times\vec{E}$$

Proof $\vec{E} = \vec{E_0}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$E_x = E_{0x}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$E_y = E_{0y}\, e^{i(\vec{k}\cdot\vec{r})-\omega t}$$

$(\vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z)$

$$\frac{\partial E_y}{\partial z} = E_{0y}\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}(ik_z)$$

$$= E_y(ik_z)$$

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$$\vec{\nabla}\times\vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

$$\frac{\partial E_z}{\partial y} = E_z(ik_y)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$(\vec{\nabla}\times\vec{E})_x = \hat{i}(ik_yE_z - ik_zE_y)$$

$$(\vec{\nabla}\times\vec{E})_y = \hat{j}(ik_zk_x - ik_xE_y)$$

$$(\vec{\nabla}\times\vec{E})_z = \hat{k}(ik_xE_y - ik_yE_x)$$

$$\vec{\nabla}\times\vec{E} = i(\vec{k}\times\vec{E})$$

Similarly $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

$$ik\times\vec{B} = i\omega\vec{E}$$

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\frac{\partial\vec{E}}{\partial t} = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}(i\omega)$$

$$= \vec{E}\,i\omega$$

$$\vec{k}\times\vec{B} = \vec{E} \Rightarrow \vec{E}\perp\vec{B}$$

$\vec{E}$, $\vec{B}$ and $\vec{k}$ are $\perp$ to each other

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Poynting vector ($\vec{P}$)

It is defined as energy flow per second per unit area held perpendicular the direction of flow or propagation.

Consider a plane electromagnetic wave travelling in vacuum along X-axis. Let electric field is oscillating along Y axis and magnetic field along Z axis. Then vector form of E and B are given

$$\vec{E} = \hat{j}\, E_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \varepsilon_0}} - kx\right] \quad \text{--- (1)}$$

$$\vec{B} = \hat{k}\, B_0 \sin\left[\frac{kt}{\sqrt{\mu_0 \varepsilon_0}} - kx\right] \quad \text{--- (2)}$$

$$\vec{E} \times \vec{B} = \hat{i}\, E_0 B_0 \sin^2\left[\frac{kt}{\sqrt{\mu_0 \varepsilon_0}} - kx\right]$$

$$= \hat{i}\, \frac{E_0 B_0}{2} \left\{1 - \cos\left(\frac{2t}{\sqrt{\mu_0 \varepsilon_0}} - 2kx\right)\right\}$$

The average value of $\langle \vec{E} \times \vec{B} \rangle$ over very large number of cycles of oscillation of $\vec{E}$ or $\vec{B}$ is given as

$$\langle \vec{E} \times \vec{B} \rangle = \frac{1}{nT} \int_0^{nT} \vec{E} \times \vec{B}\, dt$$

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(Where n is very large integer)

$$= \frac{1}{nT} \hat{i} \frac{E_0 B_0}{2} [nT - 0]$$

($\because$ Cosine function is cyclic so its average value over any number of complete cycles is zero)

$$= \hat{i} \frac{E_0 B_0}{2} \qquad (\because E_0 = cB_0)$$

$$= \hat{i} \frac{c B_0^2}{2} \text{ ——— } (3)$$

We know that $\vec{B} = \mu_0 \vec{H}$

$$\langle \vec{E} \times \mu_0 \vec{H} \rangle = \hat{i} \frac{c B_0^2}{2}$$

$$\langle \vec{E} \times \vec{H} \rangle = \hat{i} c \left(\frac{B_0^2}{2\mu_0} \right) \text{ ——— } (4)$$

But $c\left(\frac{B_0^2}{2\mu_0}\right)$ is the intensity of plane wave i.e energy flowing per unit area per second along the direction of motion of electromagnetic wave. Here by definition it is equal to Poynting vector. Thus eq^n (4) can be written as

$$\vec{P} = \langle \vec{E} \times \vec{H} \rangle$$

Hence proved